ETP 4241 – POWER SYSTEMS & ENERGY CONVERSION

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Chapter 1 - Units & Vectors

1.1 Definitions and Units

Some of the quantities commonly used in Power Systems are defined briefly as follows

Current: Rate of change of charges passing through cross-section of a conductor is current. Unit of current is *Ampere* (A)

$$i = \frac{dq}{dt} \tag{1.1}$$

In circuit analysis and power systems, most of the time current value is calculated using Ohm's law.

Potential Difference: Potential difference is the work done on a unit positive charge to displace it from one point to another against the direction of the electric field. If the initial point is located at infinity (far away that electric field is negligible at that point), potential of that point is assumed to be zero (say, ground), hence work done in moving that charge to the final point within the electric field is simply the *potential* or *voltage* of the final point. Unit of potential difference or voltage is *Volt (*V*)*

$$v = \frac{dw}{dq} \tag{1.2}$$

Once again, Ohm's law is more commonly used to calculate voltage or potential difference across different components in circuit analysis and power systems. Note that '*e'* is also used to express voltage in power systems.

Electric Power: Power is work done per unit time. Electrical power is the same except work is done in the electrical field. Using (1.1) and (1.2), mathematical expression for the electric power can be given by,

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi \tag{1.3}$$

Hence, electric power is the product of potential difference and current; an expression very commonly used in circuit analysis. Unit of power is *Watt* (W)

Force: Force is a quantitative description of the interaction between two physical bodies, such as an object and its environment. According to Newton's law, Force is the product of mass of a body and acceleration that it acquires under the application of this force. Force is a vector quantity, unlike the three electrical quantities described above, which are scalars, i.e. they only have magnitude. Force has magnitude as well as a direction, hence, a vector quantity.

$$\mathbf{F} = m\mathbf{a} \tag{1.4}$$

Note that vector quantities (as well as phasor quantities later in this text) are expressed in ***bold*** letters. Unit of Force is *Newton* (N). This expression of force is not used in power systems. The appropriate expressions will be defined in later chapters.

Torque: Torque is the tendency of force to move an object about an axis. It is a vector quantity as well and it is defined as the cross product (defined in the next section) between the force and moment arm vectors.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \tag{1.5}$$

where moment arm is the perpendicular distance from the axis of rotation to the point of application of force. The magnitude of torque is calculated by $rF\sin(\theta)$, where r and F are the magnitudes of moment arm and force, respectively, and θ is the angle between force and moment arm vectors. Note that $F\sin(\theta)$ may also be considered as the component of force perpendicular to the moment arm, as shown in *figure 1.1*. Direction of torque is perpendicular to both force and moment arm vectors. Unit of Torque is *Newton meter* (Nm).

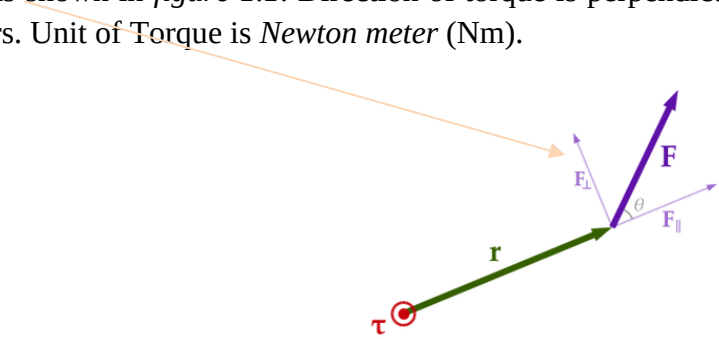


Figure 1.1: Breakdown of Force Vector into its Parallel and Perpendicular Components

Speed: Speed is defined as the rate at which some body is able to move. It is a scalar quantity. The vector counterpart of speed is *velocity*, which has speed as its magnitude as well as a direction. Unit of speed is *meter/second* (m/s)

Angular Speed: Speed of rotation, also called angular speed, is the rate at which a body is moved about an axis. Angular speed is generally represented by ω and its unit is *radian/second* (rad/sec)

Work: Work or *Energy* is the amount of force required to move an object for a distance d in the line of force. It is a scalar quantity which is the *dot product* (defined later) between the force and distance vectors.

$$W = \mathbf{F}.\mathbf{d} = Fd\cos(\theta) \tag{1.6}$$

where θ is the angle between the force and line of movement. Unit of Work is *Joule* (J)

Magnetic Flux: The number of magnetic field lines (also considered as magnetic flux density) passing through a surface. Magnetic flux is a scalar quantity which is the *scalar* or *dot* product between the *magnetic flux density* vector and *vector* representing the area of the surface (a vector perpendicular to the surface with magnitude as the area of the surface), as shown in *figure 1.2*

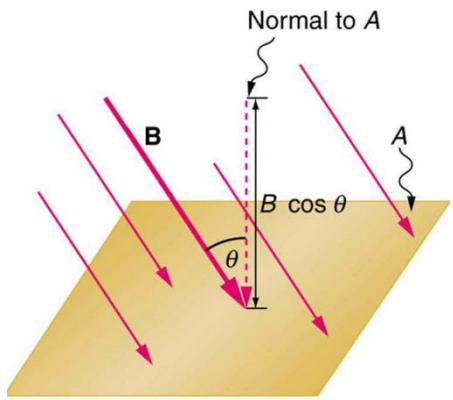


*Figure 1.2: Magnetic Flux Density (**B**) crossing a surface area (A)*

Mathematically, magnetic flux can be given by,

$$\phi = \mathbf{B.A} = BA\cos(\theta) \tag{1.7}$$

where $B\cos(\theta)$ is the component of magnetic flux density perpendicular to the surface or parallel to the surface vector. If B is passing through the surface perpendicularly such that θ is 0°, magnetic flux will be maximum and will be given simply by BA. Unit of magnetic flux is *Weber* (Wb).

Magnetic Flux Density: Magnetic Flux Density (**B**) is simply number of magnetic lines per unit area. It is a vector quantity. Unit of Magnetic Flux Density is *Tesla* (T) which is equal to Wb/m^2. Magnetic flux density will later be defined in terms of *Magnetic Field Strength*; a relationship widely used in electromagnetism to characterize magnetic materials.

1.2 Per Unit System

Standard system of measurement is *The International System of Units (SI)* system. However, there are other systems widely in use throughout the world and especially in U.S. Per Unit System is a comparative way to express values of different quantities. Advantage of using per unit system is that one can calculate the actual value of any quantity, expressed in per unit, in the desired system if the *reference* or *base* value of the quantity is known in that system.

Per Unit value of any variable can be calculated by taking the ratio of that variable with the base or reference value of the variable.

$$X(pu) = \frac{X}{X_{base}} \tag{1.8}$$

Example 1.1

If base value of voltage is 100V, what will be the per unit value of 20KV voltage?

$$V(pu) = \frac{V}{V_{base}} = \frac{20K}{100} = 200\,pu$$

Base value of a variable can also be found out from the base values of other variables from its relationship formula with those variables.

Example 1.2

If base value of voltage is 100V and base current is 10A, calculate the per unit value of 200Ω resistor.

$$R_{base} = \frac{V_{base}}{I_{base}} = \frac{100}{10} = 10\Omega$$

$$\Rightarrow R(pu) = \frac{R}{R_{base}} = \frac{200}{10} = 20\,pu$$

1.3 Vector Arithmetic

Any vector in space can be defined in Rectangular or Cartesian form by breaking it down into its components along with the three axes in space, as shown in *figure 1.3*.

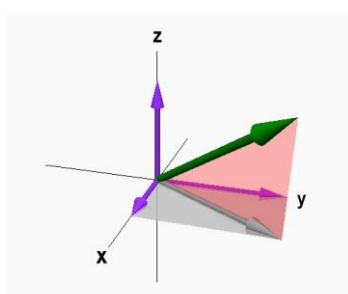


Figure 1.3: Green vector is broken down into its components along with the three axes

Hence, any vector **A** can be given in Cartesian coordinates as $\mathbf{A} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$, where a_x, a_y, and a_z are the magnitudes of the vector along *x*, *y*, and *z* axes, and **i**, **j**, and **k** are the *unit* vectors along the three axes. A unit vector is used to represent direction of a vector. A unit vector has unity

magnitude and its direction represents direction along the axis. Hence, unit vector **i** has unity magnitude with direction along *x*-axis. Same goes for unit vectors **j** and **k**, as shown in *figure 1*.

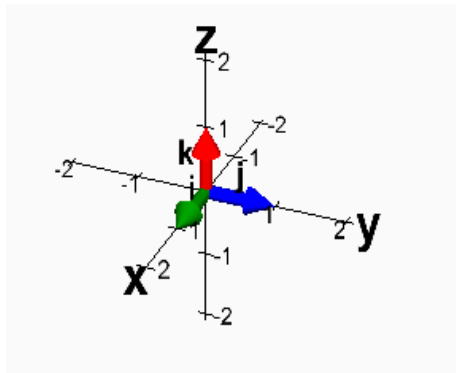


Figure 1.4: Unit vectors representing three axes

Note that throughout this text, we will be using the coordinate system as referenced in *figure 1.4*.

<u>*Dot Product:*</u> Dot product or *scalar* product is the product between two vectors that yields a scalar quantity.

$$c = \mathbf{A}.\mathbf{B} = AB\cos(\theta) \tag{1.9}$$

where A and B are the magnitudes of the two vectors **A** and **B**, and θ is the angle between them. Note that since **i**, **j**, and **k** are perpendicular to each other, hence, anytime a dot product is carried out between them, it will result in zero ($\cos(90^o) = 0$). Likewise, if a dot product is carried out between two vectors parallel to each other, say a**i** and b**i**, it will simply be ab as cos(0) is unity.

<u>*Example 1.3*</u>

Let $\mathbf{A} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ *and* $\mathbf{B} = 7\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$ *, calculate* ***A.B***

A.B = (4**i**+5**j**+6**k**).(7**i**+3**j**+9**k**) = 4**i.** 7**i** + 4**i.** 3**j** + 4**i.** 9**k** + 5**j.** 7**i** + 5**j.** 3**j** + 5**j.** 9**k** + 6**k.** 7**i** + 6**k.** 3**j** + 6**k.** 9**k**

→ **A.B =** 28+0+0+0+15+0+0+0+54 = 97

Note that **A.B** is a scalar quantity.

Dot product is a commutative process, i.e. **A.B = B.A**

<u>*Cross Product:*</u> Cross product is the product between two vectors that yields a vector quantity. The resultant vector is perpendicular to both the vectors that were involved in the cross product.

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = AB\sin(\theta)\mathbf{c} \tag{1.10}$$

where $AB\sin(\theta)$ is the magnitude of vector **C** and **c** is a unit vector which is perpendicular to both **A** and **B.** Note that if the two vectors are parallel to each other, the angle θ between them will be zero and magnitude of the resultant cross product vector will be zero as well. Also, if the two vectors are perpendicular to each other, magnitude of the resultant cross product vector will be maximum (*AB*) and direction will be perpendicular to both of the vectors.

When cross product between two different unit axes vectors is taken, it results in a unit vector along the third axes, as shown in the table below.

Going Counterclockwise	Going Clockwise
i × j = k	**i × k = -j**
j × k = i	**k × j = -i**
k × i = j	**j × i = -k**

As discussed earlier, a cross product between same unit axes vectors will result in zero, as they will be parallel to each other.

Example 1.4

Let $\mathbf{A} = 4\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$ *and* $\mathbf{B} = 7\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$ *, calculate* $\boldsymbol{A}\times\boldsymbol{B}$

A×B = (4**i**+5**j**+6**k**)×(7**i**+3**j**+9**k**) = 4**i**×7**i** + 4**i**×3**j** + 4**i**×9**k** + 5**j**×7**i** + 5**j**×3**j** + 5**j**×9**k** + 6**k**×7**i** + 6**k**×3**j** + 6**k**×9**k**

→**A×B =** 0+12**k** -36**j** -35**k** + 0 + 45**i** + 42**j** – 18**i** + 0 = 27**i**+6**j**-23**k**

Note that **A×B** is a vector quantity.

Keep in mind that cross product between two vectors **A×B** is not equal to **B×A**. Hence, cross product is not a commutative operation.

PROBLEMS

1. Three resistors have the following ratings:

Resistor	Resistance	Power
A	100Ω	24W
B	50Ω	75W
C	300Ω	40W

Use resistor *A* as your base, determine the per-unit values of resistance, power, and voltage rating of resistors *B* and *C*

2. Given flux density **B** = 4**i** + 6**j** + 7**k** wb/m^{2}over cross-sectional area of a surface given by **A =** 2**i** + 8**k** m^2, find the total flux crossing the surface.

3. Three vectors are given as follows:
   A = 4**i** + 5**j** – 6**k**
   B = 9**i** + 2**j** + 6**k**
   C = 8**i** + **9k**

   Calculate: **A×(B×C)**; (**A×B)×C; A.(B×C); (A×B).C**

Chapter 2–Electricity & Magnetism

2.1 Fourier Series Discussion

In power systems and several other engineering fields, one encounters several types of periodic waveforms. Many times these waveforms are sinusoidal but due to the addition of noise and interference, they get distorted and become sinusoidal-like periodic waveforms. Many times they are not like sinusoidal waveforms at all. For example, square waveforms, saw-tooth waveforms, triangular waveforms etc. Fourier theory explains that any non-sinusoidal periodic waveform is a combination of three components:

(i) An average or DC component, which is non-zero if the waveform has unequal areas above and below the horizontal axis.
(ii) A sinusoidal component with same frequency as the frequency of the non-sinusoidal waveform. This component is called *fundamental* component.
(iii) A number of sinusoidal components with frequencies that are multiple of the fundamental frequency. These components are called *harmonics*.

According to Fourier theory, when the above three components are added together in an infinite series, the original non-sinusoidal signal can be acquired. Mathematical expression of trigonometric Fourier series is given as follows:

$$x(t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t) \tag{2.1}$$

where a_o is the average value of the non-periodic signal $x(t)$, n is the harmonic number, ω_o is the frequency of signal $x(t)$, and a_n and b_n are the coefficients of harmonics. Note that fundamental component is the first harmonic, i.e. $n = 1$ when the frequency of sinusoidal components is equal to ω_o.

As the number of harmonic increases in Fourier series, it's contribution towards the non-sinusoidal signal decreases, i.e. it's amplitude becomes smaller and smaller. Practically, the non-sinusoidal periodic signal may be achieved if the infinite series shown is (2.1) is truncated up to the harmonic when the coefficients (amplitude) become small enough to be neglected.

Fourier series of a square waveform is given by the following expression,

$$v(t) = \frac{V_p}{2} + \sum_{n=1,3,5,7...} \frac{2V_p}{n\pi} \sin(2\pi n f_o t) \tag{2.2}$$

Assume that the peak voltage is 10V and frequency of the signal is 100Hz, if we determine the signal from (2.2) up to a specific harmonic, we will get the results as shown in *figure 2.1*.

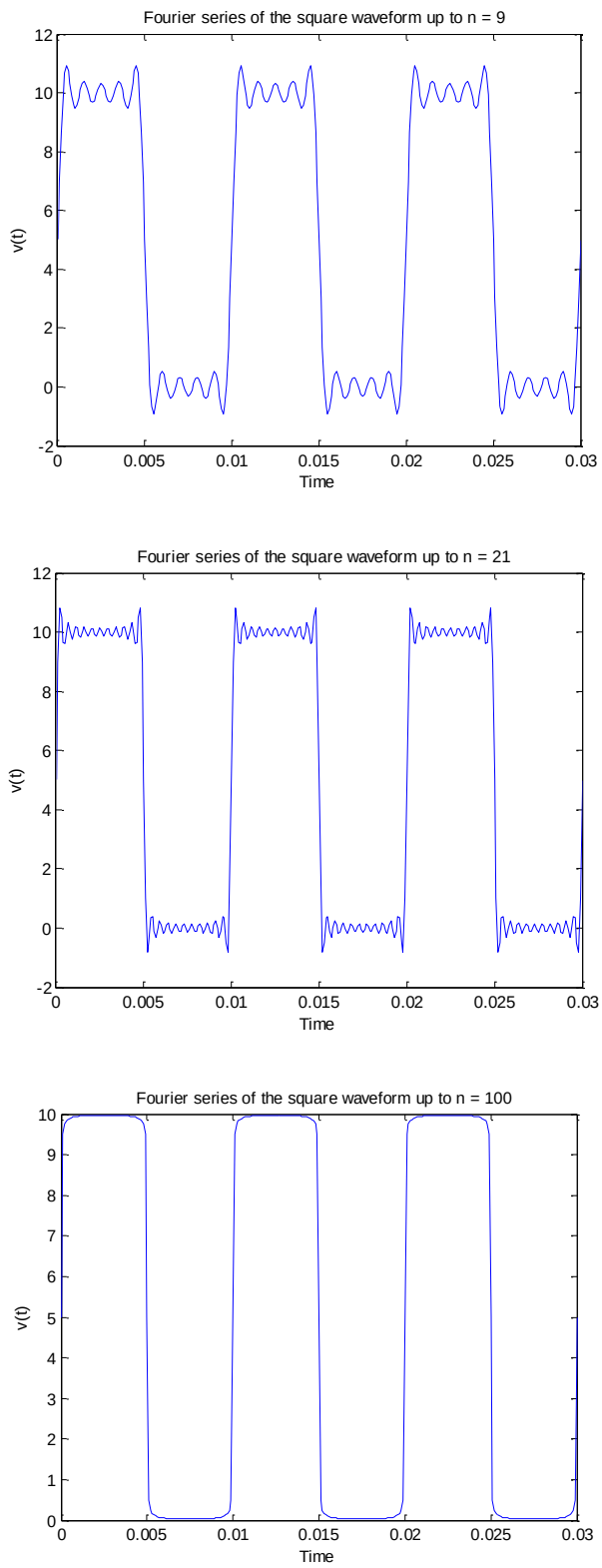


Figure 2.1: Reconstruction of a square wave through Fourier series

Example 2.1

A sinusoidal voltage signal is affected by interference and becomes non-sinusoidal due to the inclusion of harmonics. If the 13th harmonic has frequency of 223Hz, what will be the frequency of the fundamental component of the signal?

13th harmonic frequency = $13f_o \rightarrow f_o = 223/13 = 17.15$Hz

2.2 Faraday's Law of Electromagnetic Induction

Faraday's law of electromagnetic inductions states that if a time varying flux is linking a coil (inductor), a voltage is induced in the coil. This voltage is directly proportional to the rate of change of flux.

$$e_{ind} \propto \frac{d\phi}{dt}$$
$$e_{ind} = -N\frac{d\phi}{dt} \tag{2.3}$$

where constant of proportionality N is the number of turns of the coil. Observe the negative sign in (2.3). This sign is in accordance with *Lenz's* law that states that the direction of induced voltage in the coil is such that if coil is short-circuited, it will run a current in the coil that will produce a flux such that it will **oppose** the original flux that induced the voltage in the coil.

Note that if difference in flux is used instead of differential, (2.3) can be given by,

$$e_{ind} \propto \frac{\Delta\phi}{\Delta t}$$
$$e_{ind} = -N\frac{\Delta\phi}{\Delta t} \tag{2.4}$$

Faraday's law of electromagnetic induction is the basis of electric generators.

Example 2.2

A coil with 10 turns is linked with alternating flux given by $\phi(t) = 100\cos(2000\pi t)$ *mWb. Calculate the magnitude of induced voltage in the coil at t = 23ms.*

$$|e_{ind}| = |-N\frac{d\phi}{dt}| = 10(0.1)(2000\pi)\sin(2000\pi t)|_{t=23ms} = 80.03\text{pV}$$

Example 2.3

A coil with 10 turns is linked with a constant flux of 10mWb through a DC magnet. The DC magnet is now moved such that the new flux is 23mWb. It took 10ms for the flux to go from 10mWb to 23mWb. Calculate the magnitude of average voltage induced in the coil.

$$|e_{ind}| = |-N\frac{\Delta\phi}{\Delta t}| = 10\frac{(23m-10m)}{10m} = 13\,\text{V}$$

2.3 Voltage Induced in a Moving Conductor

If magnetic field is stationary but a conductor is moving such that it is cutting the field, the coil is experiencing a variable magnetic field across it. According to Faraday's law, a voltage will be induced in the coil. This voltage will depend on magnetic flux density, velocity of movement, length of conductor experiencing the linkage of flux, and it can be given by the following formula:

$$e_{ind} = \mathbf{l}.(\mathbf{v}\times\mathbf{B}) \tag{2.5}$$

where **B** is the flux density, **v** is the velocity, and **l** is the length vector. Length vector is assumed to be positive in the direction of the flow of current.

For maximum induced voltage, velocity and magnetic flux density should be perpendicular to each other, which will produce their cross product vector parallel to the length vector. In that case induced voltage will simply be *Blv*.

Example 2.4

*A conductor is moving in space with velocity 3**i**+5**j**+7**k**m/s and is placed parallel to x-axis. If length of the conductor is 1.2m and the whole length is cutting the magnetic flux given by 4**i**+8**k**T, calculate the induced voltage and its direction in the conductor.*

Assume that current is going to flow in the positive x direction, hence length vector can be taken as 1.2**i** m.

➔ e_{ind} = 1.2**i** . (3**i**+5**j**+7**k** × 4**i**+8**k**) = 1.2**i** . (-24**j**-20**k**+40**i**+28**j**) = 48V

Note that since our assumed direction of current is valid, induced voltage will be positive along positive x-axis with respect to the other side.

2.4 Lorentz Force

If a current carrying conductor is placed in magnetic field, it experiences a force. This force is called *Electromagnetic* or *Lorentz* force. Mathematically, this force can be given by,

$$\mathbf{F} = I(\mathbf{l} \times \mathbf{B}) \tag{2.6}$$

where I is the current in the conductor, **l** is the length vector of the conductor taken positive in the direction of the flow of current, and **B** is the magnetic flux density vector. Observe that the force will be maximum when length vector and magnetic flux density are perpendicular to each other. In that case magnitude of force will simply be given by,

$$F = IlB \tag{2.7}$$

Lorentz force is the basis of the working principle behind electric motors. Although direction of Lorentz force may easily be found from (2.6), there is also a commonly used method called *right-hand rule* to predict the direction of Lorentz force. According to this method, if index finger of the right hand points towards the flow of current, i.e. direction of vector **l** and middle finger points towards the magnetic flux density vector then thumb will point towards the direction of force, as shown in *figure 2.2*.

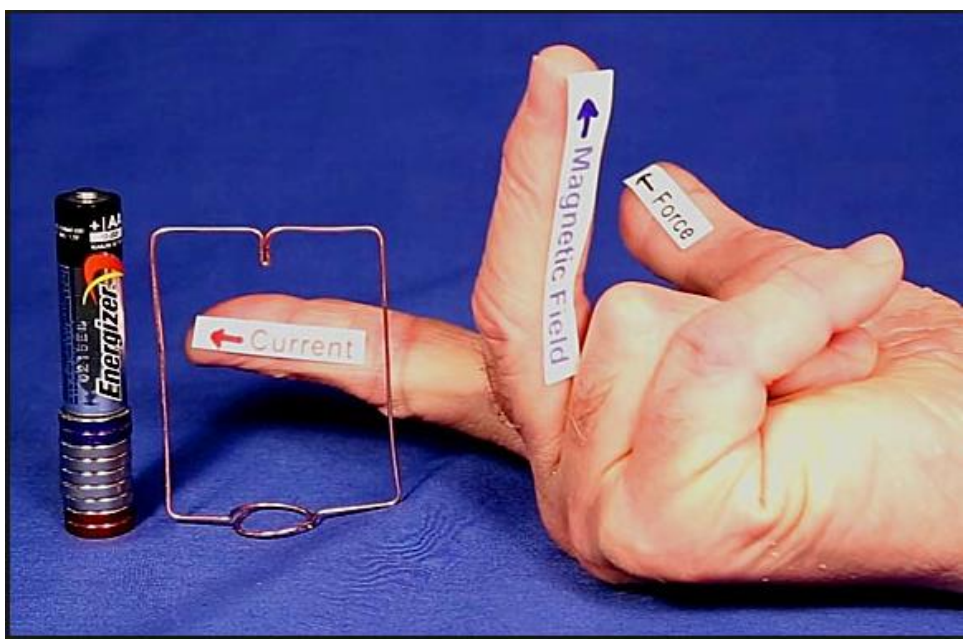


Figure 2.2: Finding direction of Lorentz force using right-hand rule

2.5 Magnetic Field Strength

Magnetic Field Strength, usually represented by variable **H**, is a vector quantity and it is another way to represent the effect of magnetic flux. Magnetic field strength is proportional to the electric current passing through a coil to produce magnetic flux. Mathematically, it is equal to the product of electric current passing through *N* turns of a coil per unit length of coil:

$$H = \frac{NI}{l} \tag{2.8}$$

Unit of magnetic field strength is *Ampere Turn/meter* (A.t/m). Magnetic field strength is also called magnetic field intensity. The product *NI* is refer to as *magnetomotive force, mmf*, which is analogous to the electromotive force, *V*, in an electric circuit. In electric circuits, one can find the potential difference (magnetomotive force) by using Ohm's law, $V = IR$, whereas in magnetic circuits, one can find magnetomotive force by using the formula:

$$\mathcal{F} = NI \tag{2.9}$$

Magnetomotive force is the property of certain substances or phenomena that give rise to magnetic fields.

Magnetic flux density is linearly related to the magnetic field strength in insulators and non-magnetic materials (including conductors like copper). Mathematically,

$$\begin{aligned} \mathbf{B} &\propto \mathbf{H} \\ \Rightarrow \mathbf{B} &= \mu \mathbf{H} \end{aligned} \tag{2.10}$$

where μ is *permeability* of the material, which is the degree of magnetization of a material in response to a magnetic field. Usually permeability of different materials is measured with respect to the permeability of vacuum, which is taken as a reference,

$$\mu = \mu_o \mu_r \tag{2.11}$$

Where μ_o is the permeability of vacuum and μ_r is the relative permeability of a material with respect to vacuum. The value of μ_o is $4\pi\times10^{-7}$ Henry/meter. Hence, for vacuum, the relationship between magnetic flux density and magnetic field strength can be given by,

$$B = 4\pi \times 10^{-7} H \approx \frac{H}{800000} \tag{2.12}$$

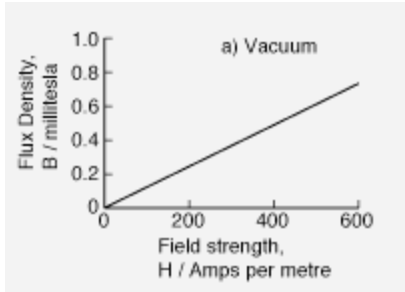


Figure 2.3: Linear relationship between B and H for vacuum. Slope of the line is 1/800000

For other non-magnetic materials (rubber, paper, copper etc.), relationship between *B* and *H* is also linear with constant values for relative permeability, hence, different slope for the straight line shown in *figure 2.3*. The linear relationship shows that the material can never be fully magnetized, i.e. the *north* and *south* poles will never be fully aligned in the material.

For *ferromagnetic* and *magnetic* materials that can be fully magnetized, the relationship between *B* and *H* can still be given by (2.9) and (2.10), i.e.

$$B = \mu_o \mu_r H \tag{2.13}$$

However, relative permeability is not constant and it changes as magnetic field strength increases and material becomes more and more magnetized. *B-H* curves for different materials are shown in *figure 2.4*

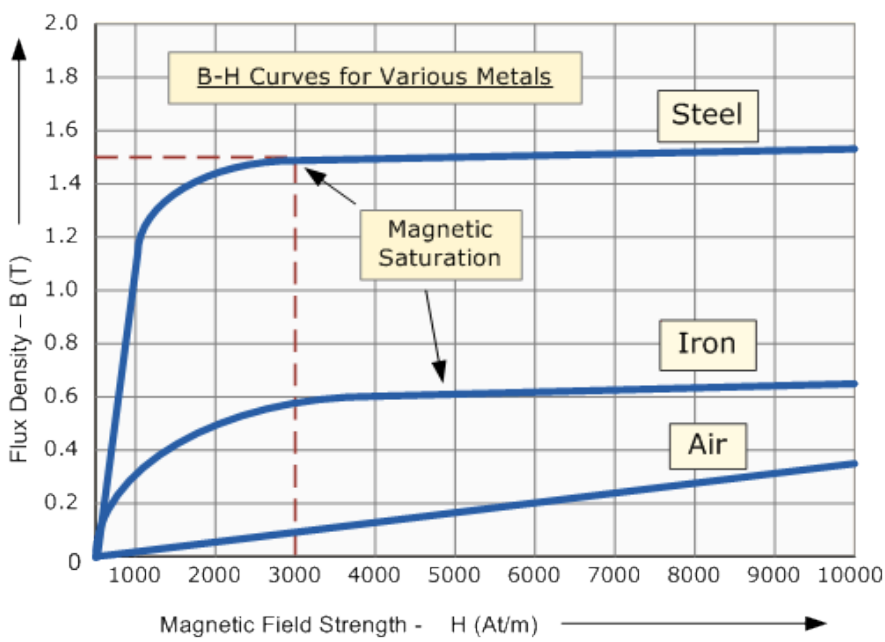


Figure 2.4: B-H relationship for different materials

Observe that while curve for *air* is linear (constant μ_r), curves for iron and steel are not linear and they saturate (magnetize) for higher values of H (variable μ_r). The value of $\mu_r$may still be calculated using (2.13)

Example 2.5

Use figure 2.4 to calculate the relative permeability for iron at 5000 At/m.

At H = 5000 At/m, B = 0.6T, hence, relative permeability will be,

$$\mu_r = \frac{B}{\mu_o H} \approx \frac{800000B}{H} = 96\,\text{H/m}$$

Hence, iron is 96 times more permeable than vacuum at H = 5000 At/m

2.6 Hysteresis Loop and Hysteresis Loss

When H increases due to flow of current in a magnetic or ferromagnetic material, magnetic poles start aligning and B increases. If current keeps on increasing, magnetic flux density saturates (all poles are aligned) as shown in *figure 2.4*. If the current that is producing the magnetic field is removed, one will assume that the curve will go back to origin and all the poles will be misaligned, leaving no magnetic flux in the material. However, this is not the case. Even when the current is removed and H goes down to zero, there is still some magnetism left in the magnetic material, as shown in *figure 2.5*.

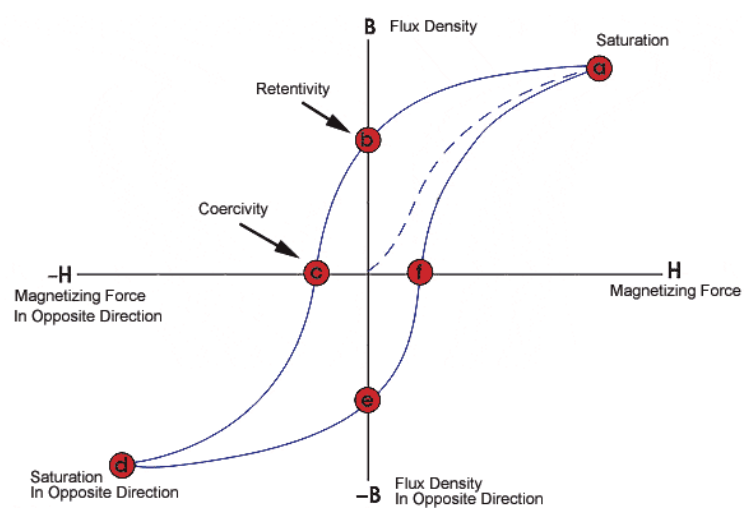


Figure 2.5: Hysteresis Loop

As it can be seen from *figure 2.5*, point *a* is the saturation point when current is increased and in turn magnetic field strength is increased, when *H* goes to zero, *B* doesn't go down to zero, it goes to point *b*. This shows that there is still some *residual flux* (B_r) left in the magnet (this property, *retentivity*, is used in shunt generators and motors). To misalign all the magnetic poles and to make the magnet neutral again, current needs to be supplied in the opposite direction, which will change the direction of *H* and finally *B* will go down to zero at point *c*. The magnetizing force required to make the magnet neutral again is called *coercive force*, H_c. If magnetic field strength keeps on increasing in the opposite direction, magnet will saturate in quadrant 3, as shown in the figure. Once again, when *H* goes to zero (current goes to zero), there is some residual flux left in magnet in the opposite direction (point *e*). To make the magnet neutral again, coercive force is required in the opposite direction, which is equal to the value of *H* at point *f*. If *H* keeps on increasing, magnet will again saturate and will go to point *a*, hence completing the loop. This loop is called *Hysteresis loop* and it is very commonly used to understand the magnetizing characteristics (retentivity, coercivity etc.) of a material. *Figure 2.6* shows the hysteresis loop in a bit more detail.

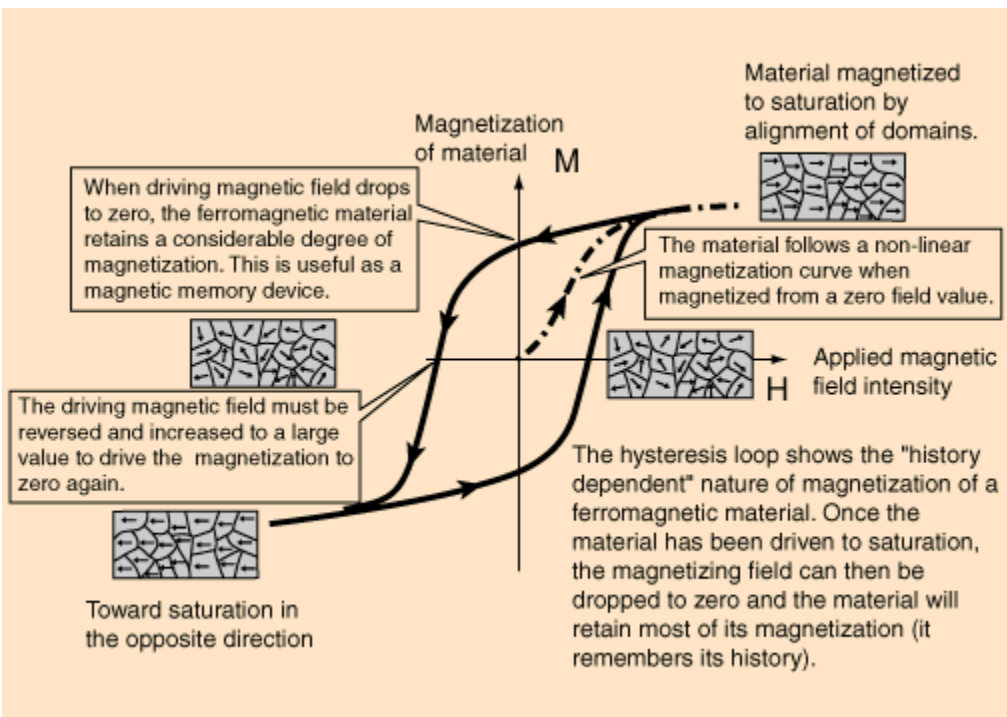


Figure 2.6: Hysteresis loop with details

In most of the electric machines including motors, generators, transformers etc., there is AC current that changes its direction. Most of the machines have their cores on which coils are wounded made out of ferromagnetic or magnetic materials. When current flows in these coils, it sets up a magnetic field in the core. Since current is alternating, magnetic field is also alternating and it can be defined by hysteresis loop. Since energy is required to set up a magnetic field and to keep changing the direction of magnetic field in the core, this energy heats up the core. This loss of energy in terms of heat is called *hysteresis loss*. To reduce hysteresis losses, cores of AC machines and equipment are made of materials with *narrow* hysteresis loops.

2.7 Eddy Currents

If an AC flux is linked with a loop of conductor, voltage is induced in it per Faraday's law. If the coil is short-circuited, an alternating current will flow through it producing heat due to coil resistance. Likewise, if another coil is placed inside the first coil, it will interact with less flux; hence, smaller voltage will induce in it which will circulate smaller amount of current if it is short-circuited as well. This scheme can be repeated several times to get circular patterns of short-circuited currents producing substantial amount of heat, as shown in *figure 2.7*

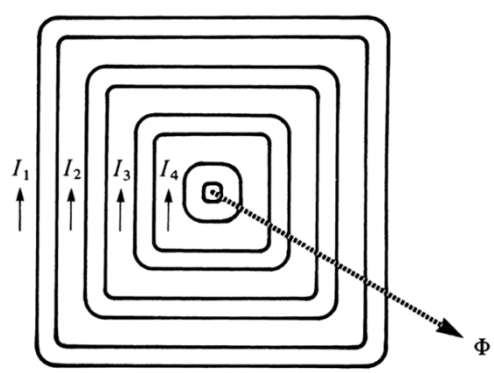


Figure 2.7: Currents revolving around short circuit loops due to induced voltages

Now consider a metal plate linked with an alternating flux. This metal plate may be considered as many short-circuit loops one inside another. Therefore, an alternating flux will induce voltages in the metallic plate which will result in the establishment of circular current paths, as shown in *figure 2.8*. These currents are called *Eddy* currents and they result in producing substantial amount of heat in metallic components linked with alternating flux.

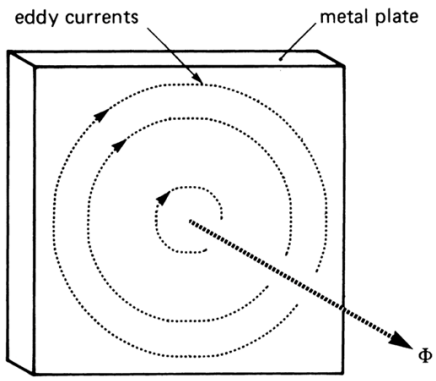


Figure 2.8: Eddy Currents

This poses a serious problem in AC motors, generators and transformers; AC flux produced due to the flow of AC current in the coils is linked with the core and other metallic parts of the machine, inducing voltage and producing Eddy currents. This result in heating of the core, termed as *Eddy current loss*, and may reduce performance of the machine as well as its life.

To alleviate this problem up to great extent, all the AC machines have cores made out of laminated sheets of metal which are insulated from each other, as shown in *figure 2.9*. This arrangement results in small induced voltages in each laminated sheet, which produce very small Eddy currents in each lamination. These small Eddy currents produce extremely small Eddy current loss in each lamination and even their cumulative effect is only a fraction of loss occurs in non-laminated core.

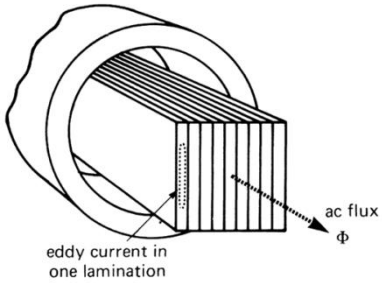


Figure 2.9: Laminated core of electric machine

PROBLEMS

1. *B-H* curves of steel, iron and air are given in *figure 2.4*. Find the value of relative permeability of steel and iron at H = 3000 At/m.

2. If a time varying flux given by $\phi = 2t^2 \cos(t)$ is linking a coil with 100 turns, what will be the magnitude of voltage in the coil at t = 2s?

3. If a conductor is cutting a magnetic field with **B** = 4**i**+ 5**k**wb/m^{2}and velocity of rotation at a certain time can be given by **v** = 2**i** + 3**j**m/s**,** what will the value of induced voltage if length of the conductor is 1.5 m and it is placed along *x*-axis? If you short-circuit the conductor, in which direction current is going to flow through it?

4. If a current carrying conductor is placed in the magnetic field as shown in *figure 2.10*, what will be the magnitude and direction of the force it will experience?

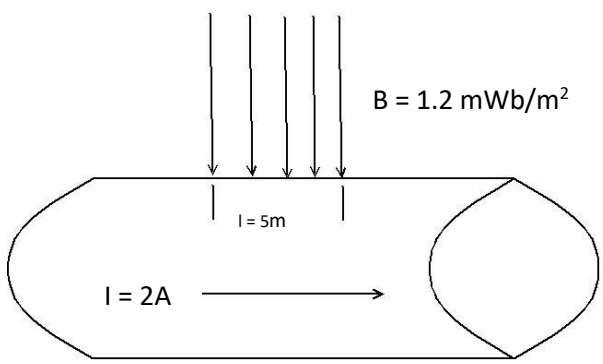


Figure 2.10: Current Carrying Conductor in a Magnetic Field

5. A single conductor is rotating counter-clockwise around the axis as shown in *figure 2.11*. Magnitude of rotation velocity is 2m/s. How much voltage will be induced at the ends of conductor (ab) if position of conductor is as shown in the figure? Also mention what will be the polarity of the two ends.

   (Hint: use equation E = (**v** x **B**)**.l** and calculate voltage on each side before adding them in series)

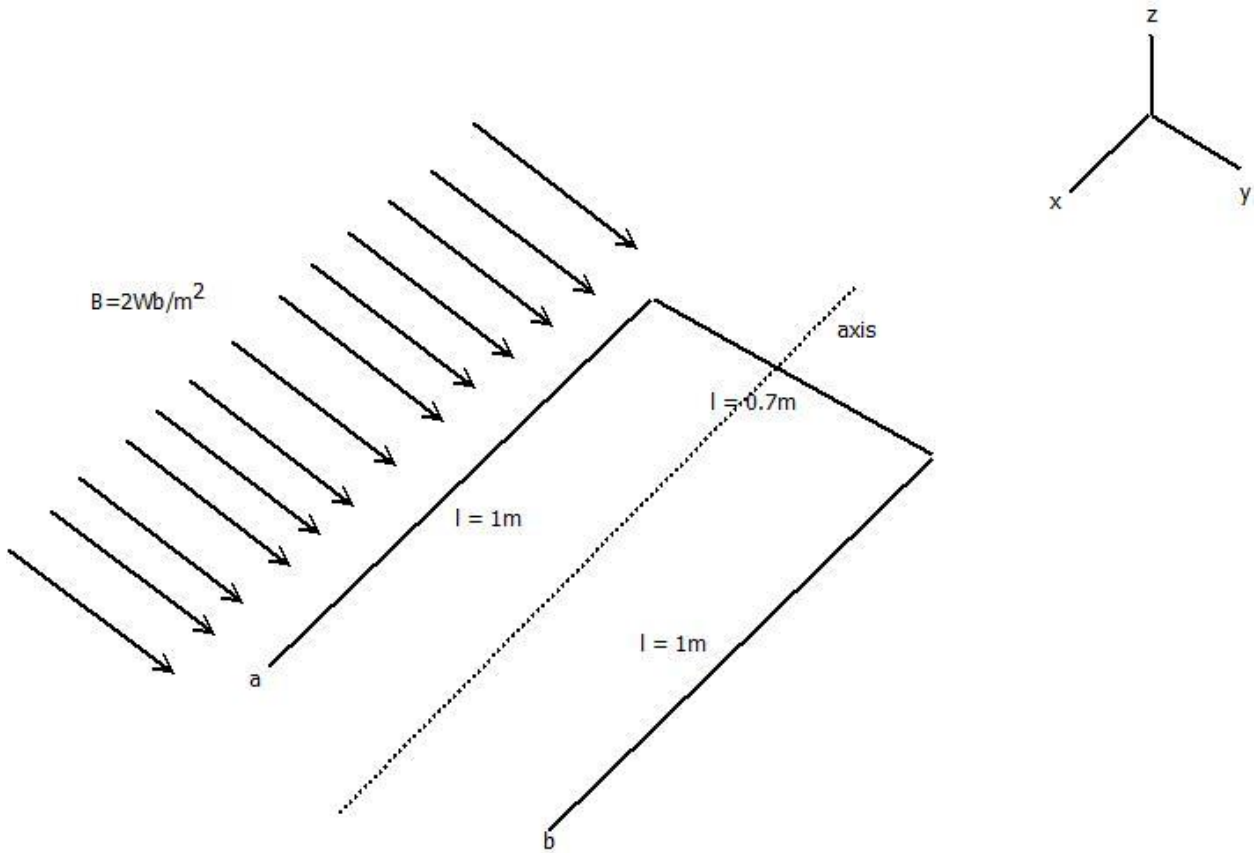


Figure 2.11: A single loop rotating inside a magnetic field

6. A time dependent flux is given by $\phi(t) = 5t^3\ln(t)\sin(3000t)$ Wb is linked by a coil with 14 turns. Determine the **magnitude of the induced voltage** in the coil at ***t* = 145ms.** Show all of your work.

7. A 2m conductor is placed along *x*-axis and moving in the space with the velocity 5**i**+6**j**+7**k** mm/μs. A magnetic flux density **B** = 3**i** +7**j** mWb/m^2 is acting on the conductor. Determine the value and direction of the induced voltage in the conductor.

8. A 10cm long conductor is placed along *y*-axis and moving along *x*-axis in the negative direction with a speed of 230cm/ms. It is linked with a flux density of 2.5mWB/m^2, which is along the positive *z* direction. Determine the voltage induced in the conductor and its polarity.

9. A time varying flux $f(t) = 2t^2\cos(2000pt+55^o) + \sin(6000pt+43^o)$ Wb is linked with 12 turns of a coil. Determine the magnitude of the voltage that will be induced in the coil at 5.3ms. Show all of your work.

10. A 200cm long wire is carrying 10A of current in the +z direction. A magnetic flux density $3\mathbf{i}+5\mathbf{j}+7\mathbf{k}$ Wb/m^2 is acting on the coil. Determine the force that the coil will experience.

Chapter 4 – Direct-Current Generators

4.1 Principle of Generator Operation

Electrical generators are operated on the principle of Faraday's law, i.e., if a loop of wire is linked with a time-varying magnetic flux, a voltage will be induced in the coil. If a load is connected to the terminals of the coil, this induced voltage will drive a current through the load. Based on this explanation, observe the arrangement of a basic electric generator in *figure 4.1*.

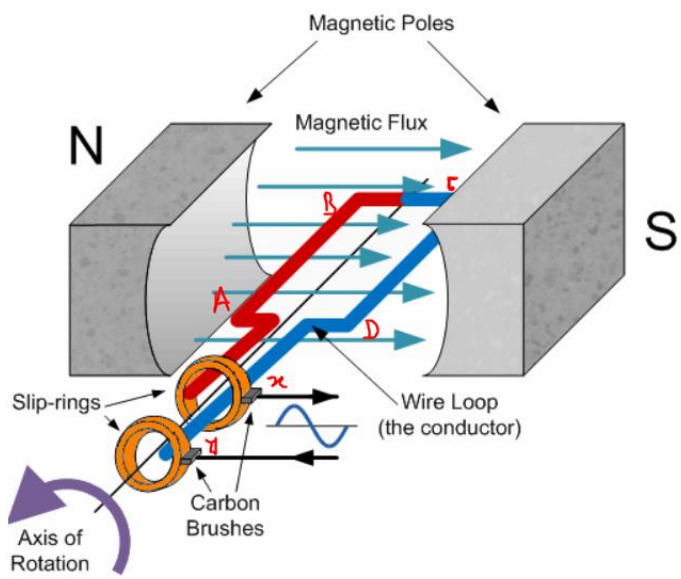


Figure 4.1: Basic Alternator (A.C. Generator)

As it can be seen in *figure 4.1*, different parts of a generator are *magnets* to produce magnetic field, a *coil* that is rotating in the magnetic field in which voltage will be induced, two *slip rings* where individual conductors from each end of the coil are connected, and two *carbon brushes* that carry current to the load. The assembly of the coil where voltage is induced in the machine is called *armature*. Most of the machines have rotating armature but there are some machines where magnetic field is rotating and armature is stationary.

To understand how the voltage will be induced in the coil, observe that the magnetic field **B** on the coil is from left to right (**j** direction), assume that axis of rotation is right in the middle of the coil along *x-axis*, the left half of the coil (red) will have velocity vector going downwards (**-k** direction) and the right half (blue) will be going upward (**k** direction). The voltage induced on the side under North Pole of the magnet (*AB*) will be along the positive *x* direction [e_{ind} = **l.(v×B) =**

i.(-k×j)], and under the South Pole of the magnet*(CD)* will be along the negative *x* direction [e_{ind} = **l.(v×B) = i.(k×j)]**. Observe that the two sides that are parallel to the magnetic field (sides *AD* and *BC*) will have no voltage induced in them as the length vector will be along *y*-axis (**j** or **–j)** and **v×B** will yield a vector along **i** or **–i**. Hence, the length vector and the resultant vector will be perpendicular to each other, and their dot product will be zero. Total voltage induced in the coil will be $2e_{ind}$, as shown in *figure 4.2*

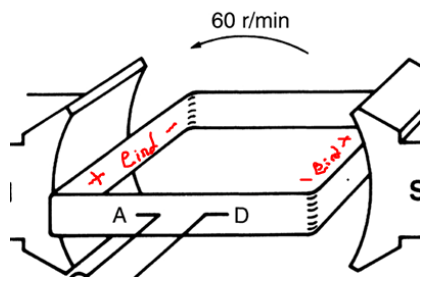


Figure 4.2: Voltage induced in the rotating coil

Now, if the coil is turned 90°, the velocity vector and magnetic field vector will be parallel to each other. Hence, **v×B** will be zero and voltage induced will be zero as well. If the coil further turns 90° (total 180°), the side that was under North Pole is now under South Pole and vice-versa. Since the conductors *A* and *D* are still touching the same slip rings and carbon brushes *x* and *y*, the voltage induced between *x* and *y* will be negative ($-2e_{ind}$). When coil further turns 90°, velocity vector will again be parallel to the magnetic field vector, inducing zero voltage in the coil. One more 90° rotation will bring the coil in its original position, as shown in the figure, making the voltage induced between *x* and *y* to be $2e_{ind}$ again. This process will keep on going which will yield an alternating shape of the induced voltage as shown in *figure 4.3*. Note that it is assumed that e_{ind} is 10V in the following figure, hence $2e_{ind}$ is 20V.

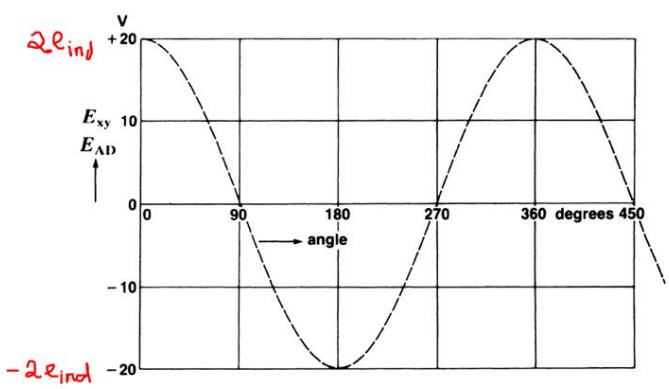


Figure 4.3: Induced voltage in the loop as a function of angle of rotation

Figure 4.4 shows the induced voltage plot against time assuming speed of rotation to be 60 rpm. There will be one rotation per second giving frequency of the rotation to be 1Hz and time period of the waveform to be 1 second.

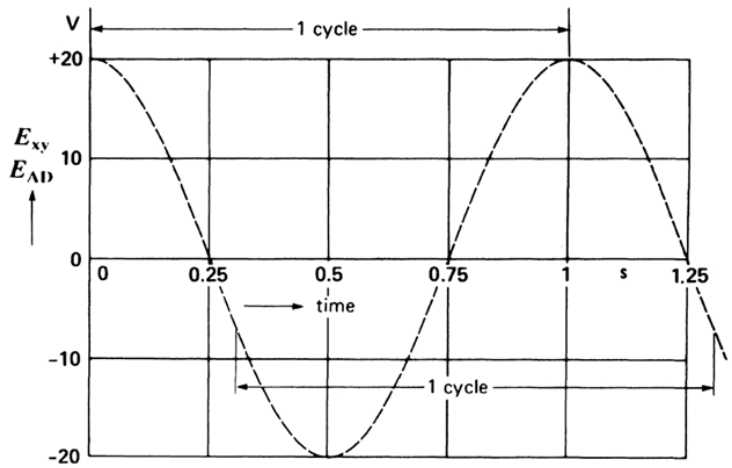


Figure 4.3: Induced voltage in the loop as a function of time

Note that the basic generator does not always produce a pure sinusoidal voltage as shown in the above figures. The shape of the output voltage depends upon the shape and enclosure of the magnets. If properly designed, shape will be very close to sinusoid else there will be harmonics in the induced voltage. For the sake of simplicity, for our study, most of the time we will assume a sinusoidal induced voltage.

4.2 <u>Commutation Process</u>

As explained in the previous section, the natural shape of the induced voltage in a rotating coil is bipolar, i.e., a coil rotating in a magnetic field will always have an AC voltage induced between its terminals. To get a unipolar (DC) voltage, a special arrangement is done at the terminals as shown in *figure 4.4*. The two slip rings are replaced by a single one which is split in two halves insulated from each other. This is called a *commutator*. The two carbon brushes are touching the two halves of the commutator. As the coil moves, the commutator halves keep switching between the two carbon brushes instead of touching the same one as it was the case with the arrangement shown in *figure 4.1* that produced AC voltage at the load. Now when the coil *AB* is under the North Pole, it will be touching the top brush *x*, and once it will move to the other side (under South Pole), it will be touching the bottom brush *y*. At this time coil *CD* will be under the North Pole touching the top brush *x*. Hence, the voltage at brush *x* will always be positive with respect to brush *y* throughout the course of rotation and load voltage will be *DC Pulsating* as shown in *figure 4.5*.

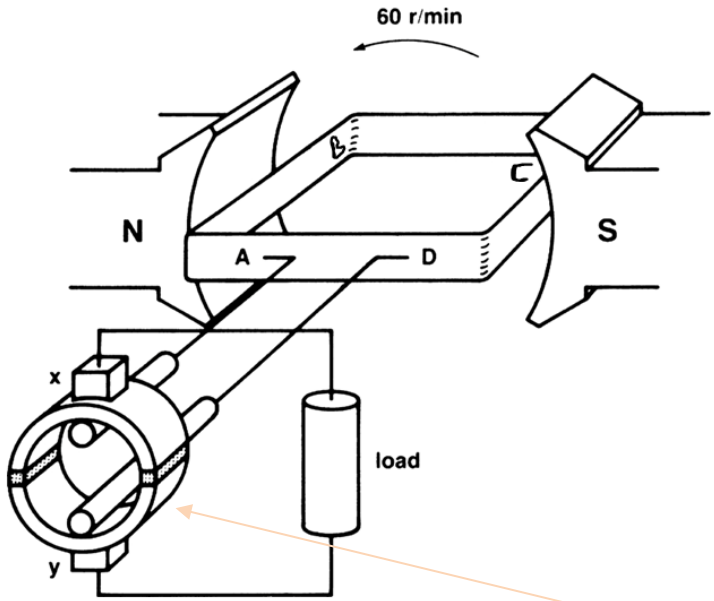


Figure 4.4: Conversion of bipolar voltage to unipolar at the load using Commutator

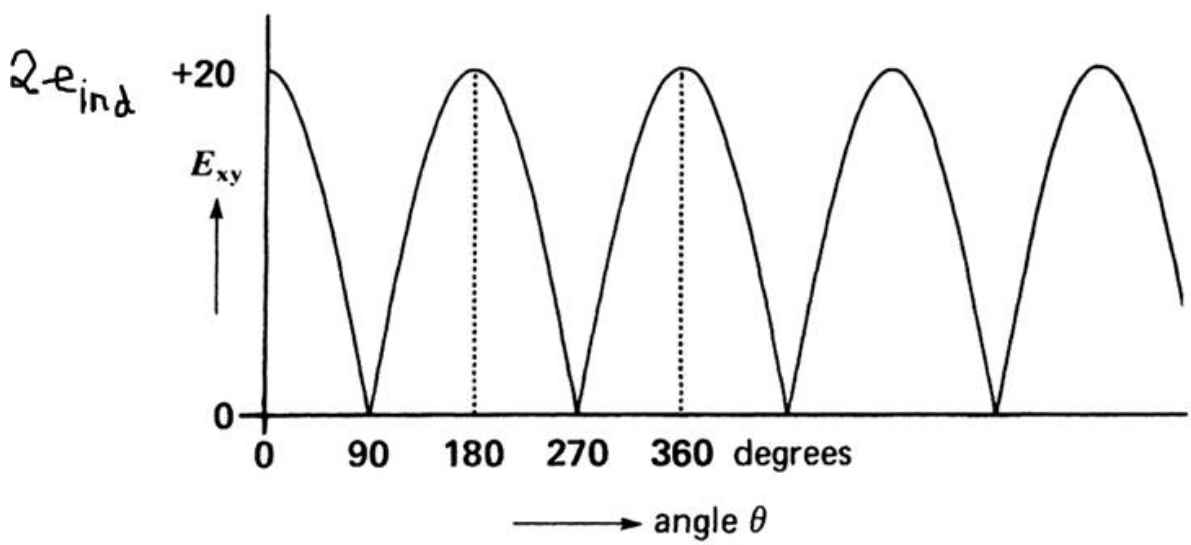


Figure 4.5: Output voltage at load

As shown in *figure 4.5*, at 0º, side *AB* is under the North Pole and *CD* is under the South Pole, inducing voltage $2e_{ind}$ in the coil which is positive from *x* with respect to *y*. At 90º, the coil is perpendicular to the magnetic field and the velocity vector is parallel to the field, hence, no voltage will be induced as **v×B** will be zero. At 180º (half turn), *CD* is under the North Pole and *AB* is under the South Pole but now commutator segment connected to *CD* is touching the brush *x* and commutator segment connected to *AB* is touching the brush *y*. Hence, voltage induced in the coil is still $2e_{ind}$which is positive from *x* with respect to *y*. Therefore, for the load connected between brushes *x* and *y*, the voltage will always be unipolar. *Figure 4.6* further explains this process.

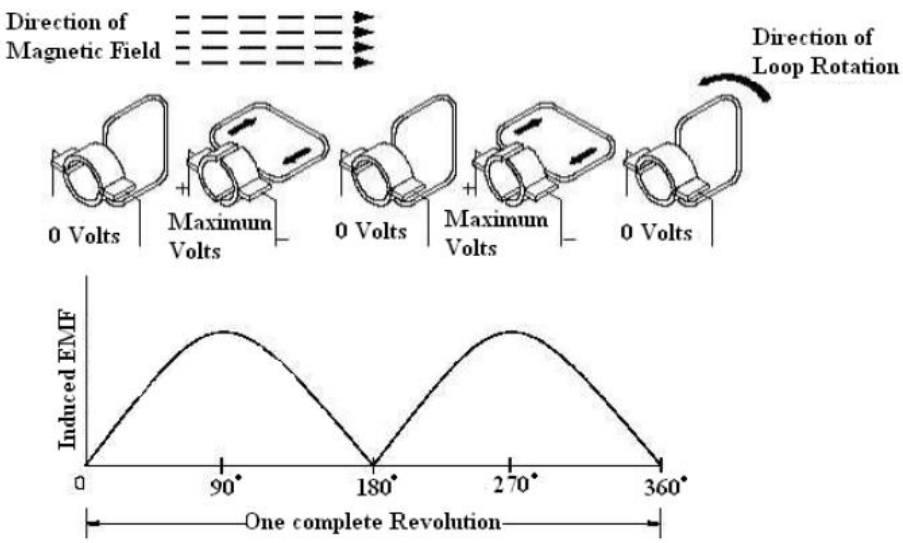


Figure 4.6: DC pulsating voltage generation using commutation process

4.3 Improving Voltage Shape

The voltage at the load using the two segments of the commutator is pulsating DC where pulses have the maximum height, i.e., from zero to maximum induced voltage, as explained in the previous section. This voltage is not very useful as a DC voltage. In practical DC generators, pulses in the output voltage are reduced by increasing the number of coils and number of commutator segments to which they are connected. Observe how the voltage is going to change if we have two coils (red and black), perpendicular to each other, and four commutator segments, as shown in *figure 4.7*. Since there are now four commutator segments and only two brushes, the voltage cannot fall any lower than point *A*, i.e., the point where the individual voltages in each coil intersect each other (dotted black lines). Therefore, the pulse or ripple is limited to the rise and fall between points A and B on the graph. By adding more armature coils, the ripple effect can be further reduced. Decreasing ripple in this way increases the effective voltage of the output.

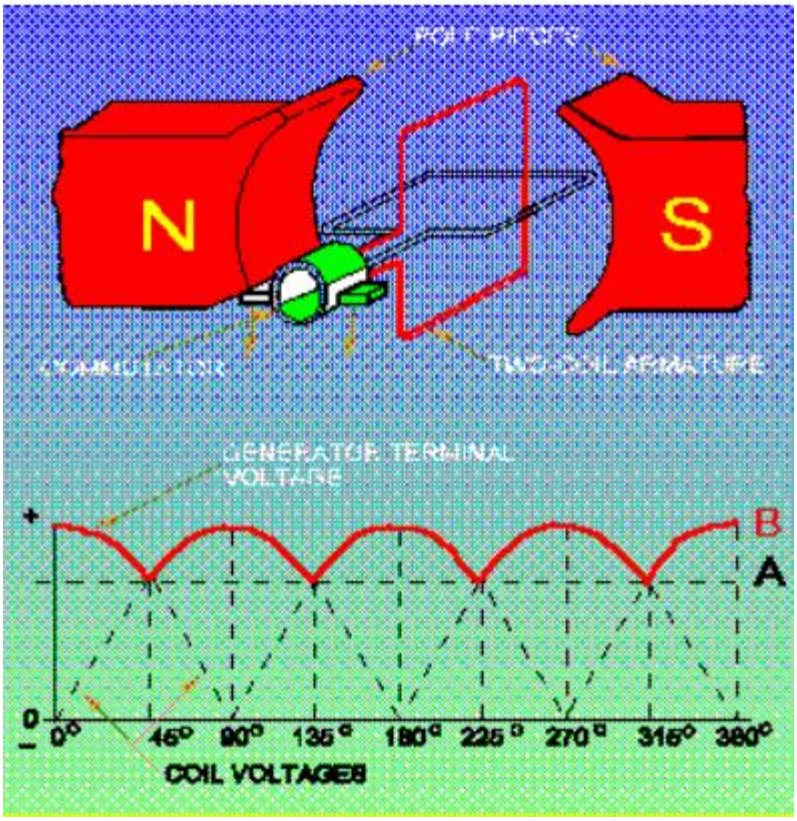


Figure 4.7: Ripple reduction at the terminal with more coils and commutator segments

By using additional armature coils, the voltage across the brushes is not allowed to fall to as low a level between peaks. Practical generators use many armature coils. They also use more than one pair of magnetic poles. The additional magnetic poles have the same effect on the ripple as the additional armature coils. In addition to that, the increased number of poles provides a stronger magnetic field (greater number of flux lines). This, in turn, allows an increase in the output voltage because the coils cut more lines of flux per revolution.

4.4 <u>Commutation in a Four Loop DC Generator</u>

Most of the armatures have either same number of coils as the number of slots or more coils fitted into lesser number of slots. Each coil can have multiple turns as well, and each turn or loop has one conductor on each side, i.e., total two conductors for one turn. Observe in *figure 4.8* how four loops are fitted into four slots.

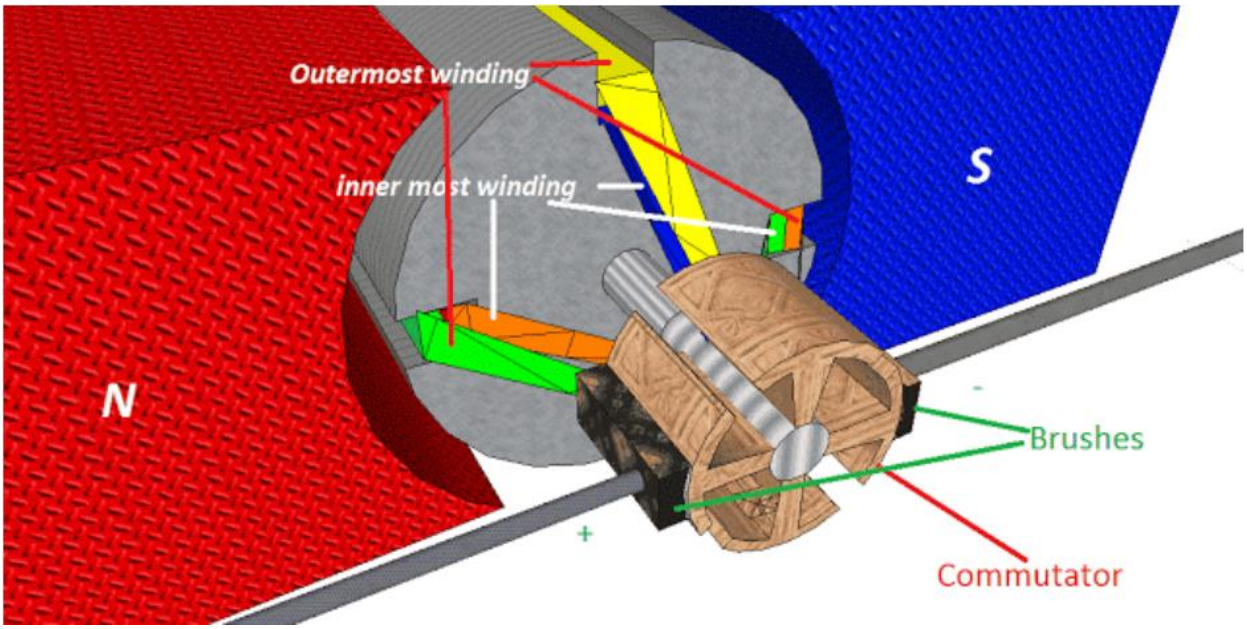


Figure 4.8: An armature with four slots, four loops and four commutator segments

In *figure 4*.8, armature of DC machine is shown placed in between the permanent poles of a magnet. Four loops with yellow, blue, green and orange color form armature winding and ends of these loops are connected with the commutator of the machine. Outermost and innermost wires of loops around the rotor are represented. The commutator is touching the carbon brushes and direction of rotation is such that polarities of brushes are '+' and '-', as shown.

Consider *figure 4.8* in 2-D at moment ωt=0 in *figure 4.9*. Four loops are shown with red, yellow, blue and orange color. The outermost wires of every loop are labeled with numbers 1, 2, 3 and 4, whereas innermost wire is labeled with prime over a number, i.e. 1′, 2′, 3′ and 4′. *a*, *b*, *c* and *d* represent four commutator segments.

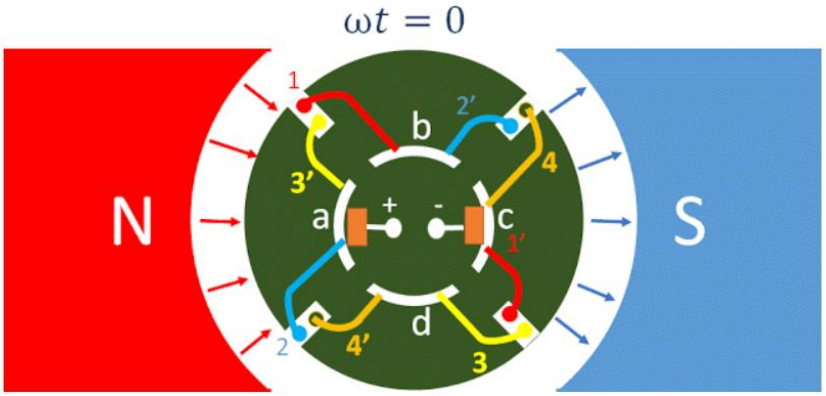


Figure 4.9: 2-D image of armature with four slots, four coils, and four armature slots

Observe that the outermost wire of the blue loop (2) is connected to commutator segment *a*, whereas innermost wire (2') is connected to *b*. Similarly outermost wire of the orange coil (4) is connected to *c*, whereas innermost wire (4') is connected to segment *d*. Red wire, 1 and 1′, are connected to *b* and *c*, whereas yellow wire, 3 and 3′, are connected to *d* and *a*respectively.Now if we draw the armature windings from *figure 4.9* in 2-D then the polarity over segments is due to connections with brushes, as shown in *figure 4.10*.

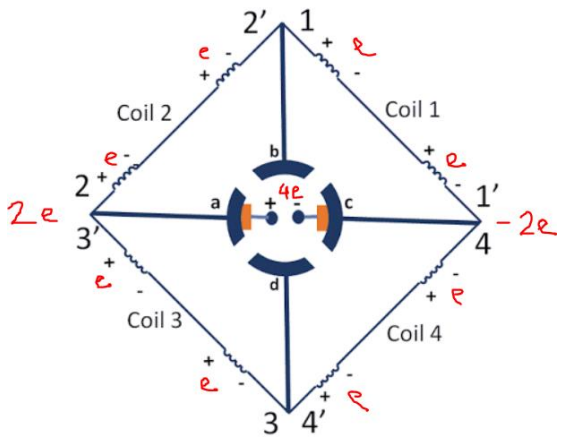


Figure 4.10: Armature coil arrangement in 2-D

Observe from *figure 4*.9 that wire terminals 1,3′,2 and 4′ are under magnetic force of the North Pole, whereas 1′,3, 2′ and 4 ends are under magnetic force of the South Pole. Assume that each loop has a voltage contribution of *e*. In the present case, all four loops are lying within poles so overall voltage is E_{ind} =4*e* between brushes *a* and*b*.

Now assume that the armature is rotated counter clockwise by 45°. Now the blue and orange loops are not facing any magnetic force, whereas red and yellow loops are under magnetic force, as shown in *figure 4.11*.

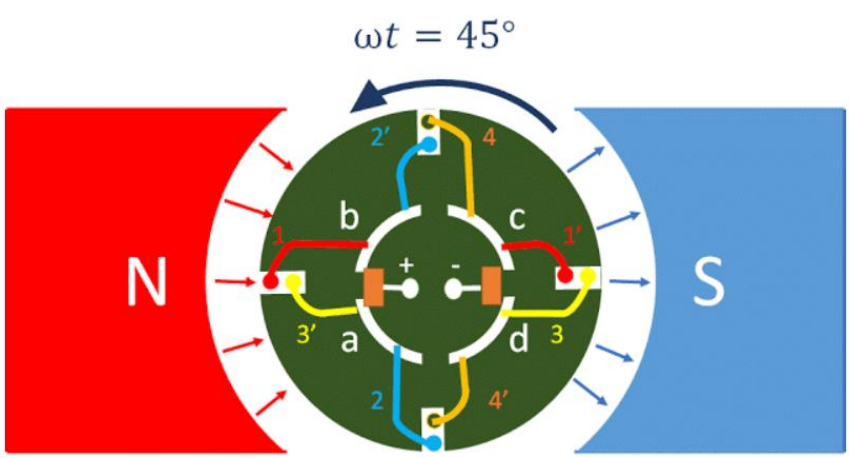
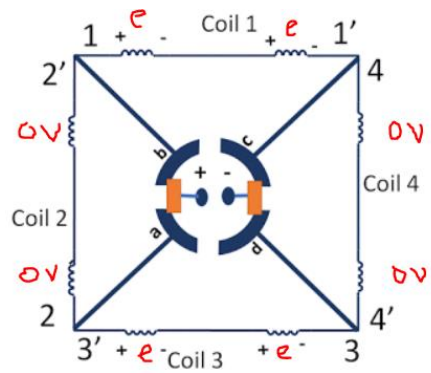


Figure 4.11: Armature rotated counter clockwise by 45°

No voltage will be induced in coils 2 (blue) and 4 (orange) at this time and voltage *e* will be induced in each of the conductor of loop 1 (red) and 3 (yellow), as shown in *figure 4.12*.



4.12: Voltage induced in the armature coil rotated to 45º

Notice that at this instant brushes of the machine are shorting out commutator segments *ab* and *cd*. This happens just at the time when the loops between these segments have 0V across them, so shorting out the segments creates no problem. At this time only loops 1 and 3 are under the pole faces, so the terminal voltage will be *2e*.

Now if the armature is rotated counter clockwise by 45º again, i.e. total 90º, it will look like the one shown in *figure 4.13*.

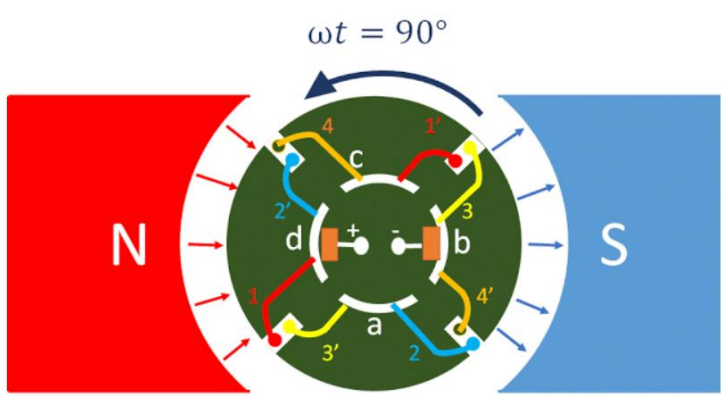


Figure 4.13: Armature rotated counter clockwise by 90º

The voltage induced between the brushes will again be *4e*, as shown in *figure 4.14*.

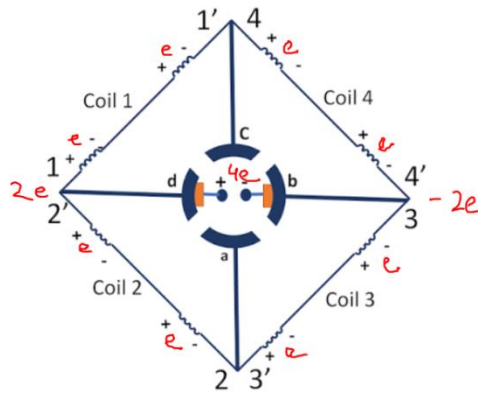


Figure 4.14: Voltage induced in the armature coil rotated to 90°

Note that the output voltage shape is similar to the one shown in *figure 4.7* except the amplitude has increased. This is due to the fact that number of coils has increased in the slots.

4.5 Types of Armature Winding

The two most common types of armature winding are *lap winding* and *wave winding*.

Lap Winding

Lap winding is the winding in which successive coils overlap each other. It is named "Lap" winding because it doubles or laps back with its succeeding coils. In this type of winding, the finishing end of one coil is connected to one commutator segment and the starting end of the next coil, situated under the same pole, is connected to the same commutator segment, as shown in *figure 4.15*

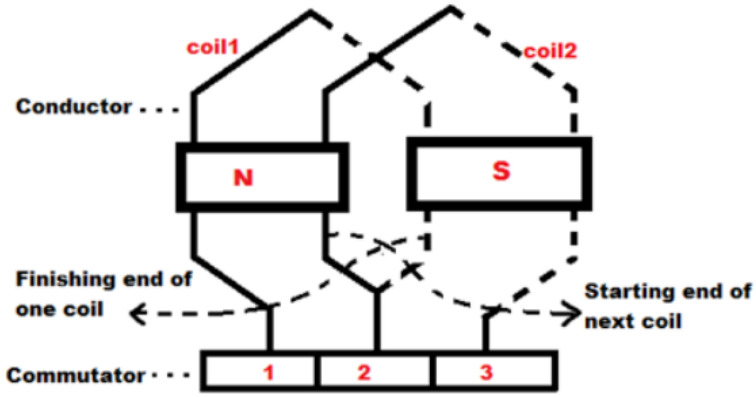


Figure 4.15: Lap Winding

As it can be seen in *figure 4.15*, the finishing end of *coil 1* and starting end of *coil 2* are both connected to the commutator segment - 2 and both coils are under the same magnetic pole that is North Pole here.

There are two types of lap windings; *simplex* and *duplex*. In simplex winding, the number of parallel paths between the brushes is equal to the number of poles, and in duplex lap winding, the number of parallel paths between the brushes is twice the number of poles. For this reason, lap winding is also called *parallel* winding. Simplex lap winding is shown in *figure 4.15* and depicted again in *figure 4.16* along with duplex lap winding.

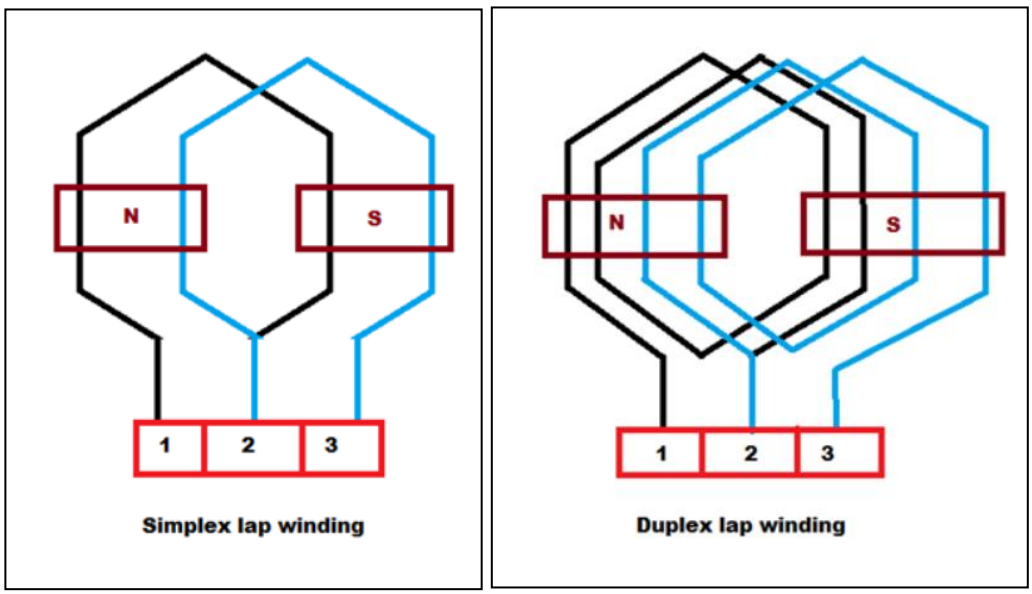


Figure 4.16: Simplex and Duplex Lap Windings

Lap winding is used in low-voltage high-current generators as there are more parallel paths. However, it gives low induced voltage for the same number of conductors as compared to wave winding. It also has less efficient utilization of space in the armature slots. Voltage induced in the armature with lap winding can be given by,

$$E_o = \frac{Zn\phi}{60} \tag{4.1}$$

where Z is the total number of conductors, n is the speed of rotation in rpm, and φ is flux per pole. Most of the study in our text is based on the assumption that armature has lap winding.

Wave Winding

In wave winding, the end of one coil is connected to the starting of another coil such that all the coils carrying current in the same direction are connected in series i.e., coils carrying current in one direction are connected in one series circuit and coils carrying current in the opposite direction are connected in other series circuit. This is why wave winding is also called *series* winding. Wave winding is shown in *figure 4.17*

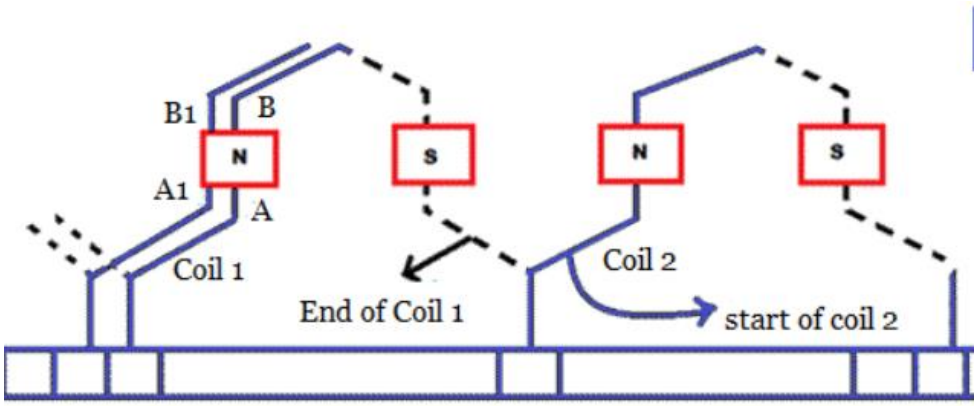


Figure 4.17: Wave Winding

If after passing once around the armature the winding falls in a slot to the left of its starting point then the winding is said to be retrogressive. If it falls one slot to the right, it is progressive. Both types are shown in *figure 4.18*

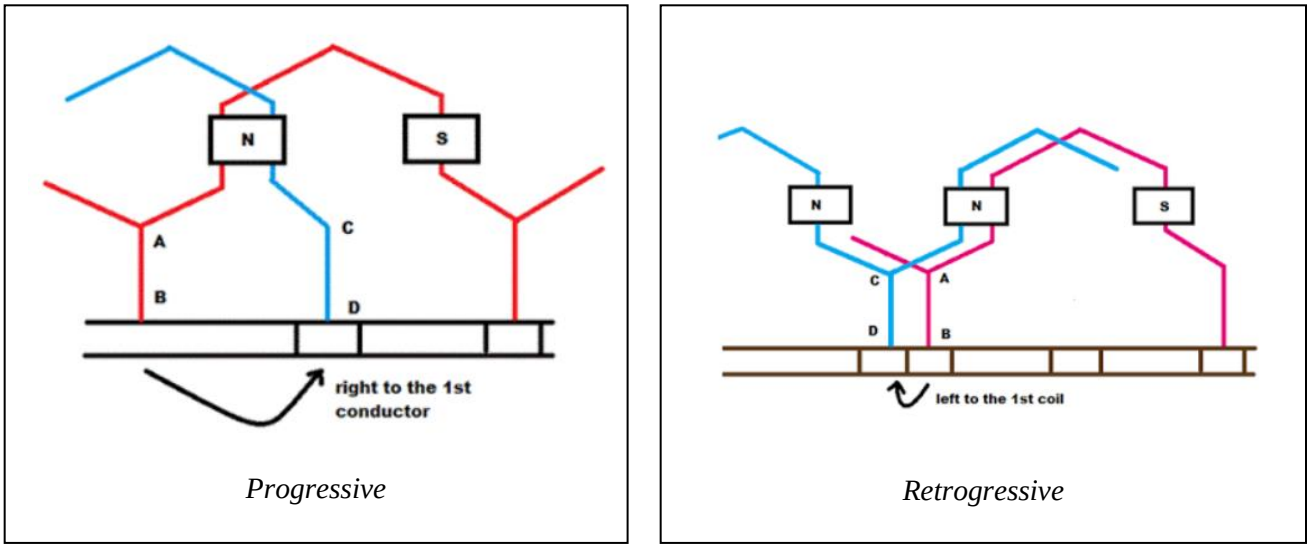


Figure 4.18: Types of Wave Winding

Wave winding is used for low-current high-voltage machines. Both lap and wave windings in an armature are shown in *figure 4.19*

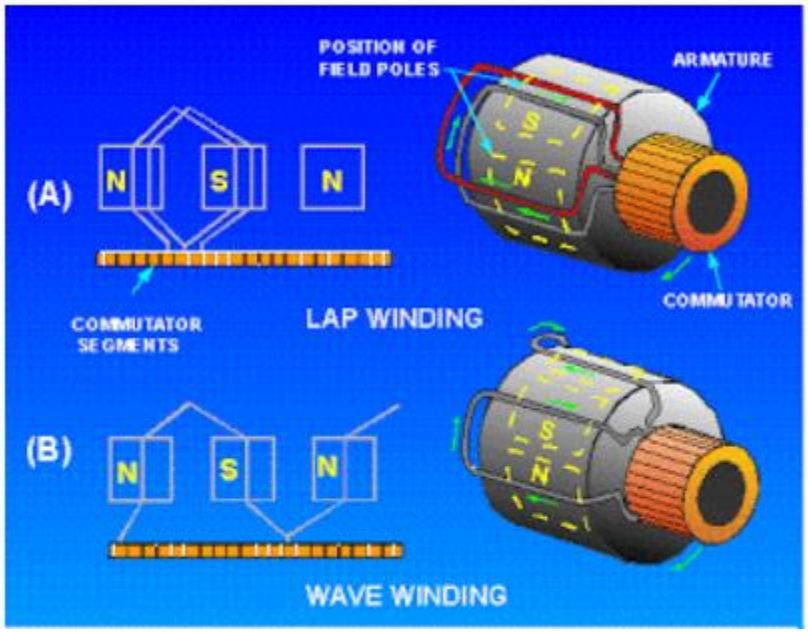


Figure 4.19: Lap and Wave Windings

A good video demonstration of the two types of windings can be found through this link.

4.6 Armature Reaction

Let's take a step back and observe *figure 4.12* once again where *coil 2* is shorting commutator segments *a* and *b* and *coil 4* is shorting commutator segments *c* and *d*. Since *coil 2* and *coil 4* do not have any induced voltage, hence, even if they short two commutator segments, it will not do any harm to the brushes connected to those segments. If there would be some voltage induced in those coils, then there would be a lot of sparking at the brushes which will lead to less than full voltage available at the brushes and constant sparking also reduces the life of the brush. Hence, it is very important to position the brushes on the armature such that if a coil shorts two commutator segments that are touching the brush, voltage induced in those segments should always be zero.

An armature with 12 coils and 12 commutator slots is shown in *figure 4.20*. Observe that the two coils that are shorting the commutator segments touching the brushes, again, have zero induced voltage as no flux is linking with them. The plane along which flux is zero is called *neutral zone* or *neutral* plane and brushes are supposed to be placed on this plane as any coil on neutral zone

will have zero voltage induced in it, hence, eliminating the danger of brush sparking and reduced output voltage.

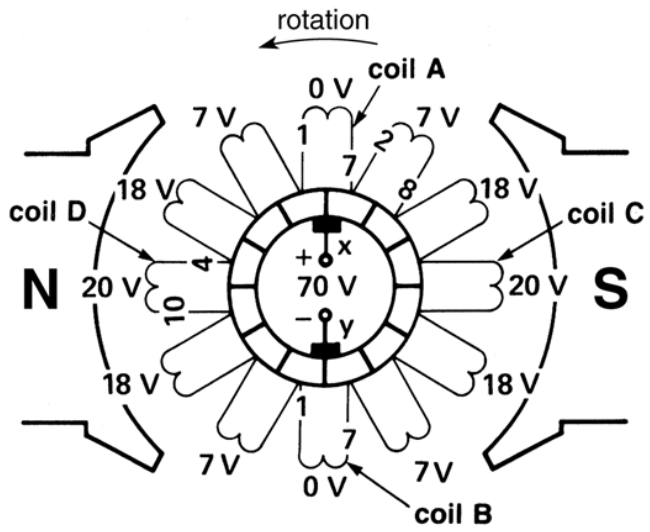


Figure 4.20: Output terminal voltage with brushes located along neutral zone

As shown in *figure 4.20*, induced voltage between the output terminals will be 70V (7+18+20+18+7). If now we connect the brushes not along the neutral zone, as shown in *figure 4.21*, observe that the terminal voltage goes down to 63V. There will also be sparking at the brushes as the two commutator segments touching the brushes are now shorted out by coils with non-zero induced voltage in them.

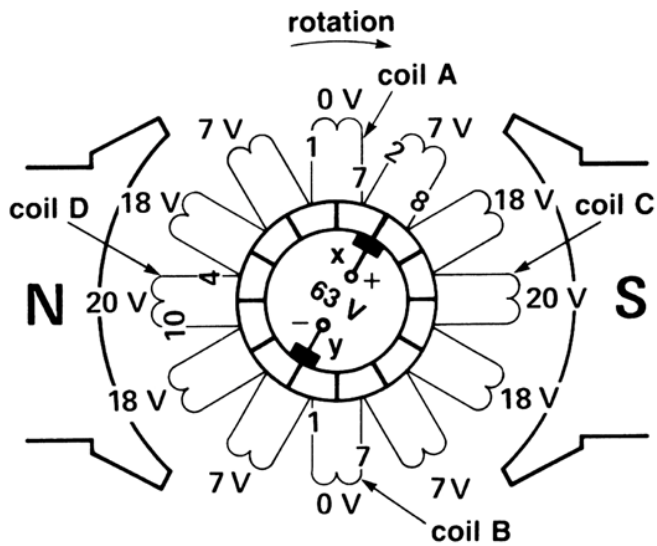


Figure 4.21: Output terminal voltage when brushes are not located on neutral zone

When a load is connected to the output terminals and current starts flowing in the armature, a magnetic field is produced due to this flow of current. This is armature magnetic field producing armature flux, as shown in *figure 4.22*. Note that '.' represents current going into the page whereas '+' represents current coming out of the page.

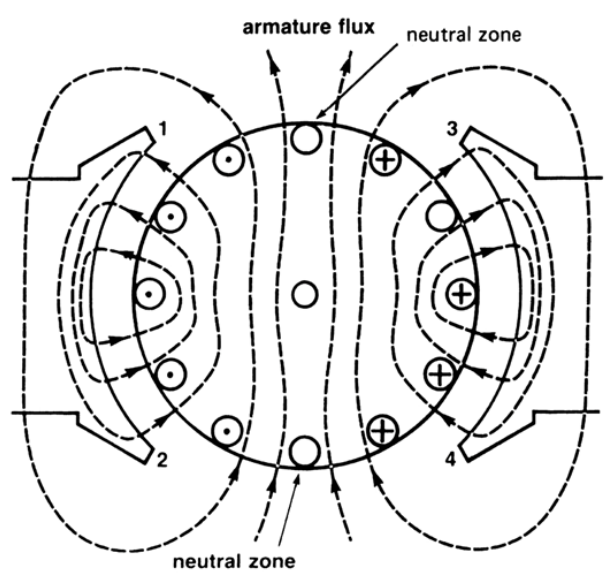


Figure 4.22: Armature flux due to armature current

When the armature flux interacts with the field flux, the cumulative magnetic flux linking the coil is changed. Now, it is quite possible that this cumulative flux is linking the coils along neutral zone and some other coils are not linked with the flux at all. Hence, orientation of the neutral zone is changed, as shown in *figure 4.23*. Since brushes are still located along the original neutral zone (neutral zone without any load connected), output voltage will be reduced and sparking will occur at the brushes. This dislocation of neutral zone is called *armature reaction* that results in reduced output voltage and brush sparking, which consequently reduces the life of brushes.

There are two ways to tackle armature reaction. The first one is to relocate the brushes along the new neutral zone. This is a temporary fix as if generator load will change, armature current will change and the new cumulative flux will change the location of neutral zone.

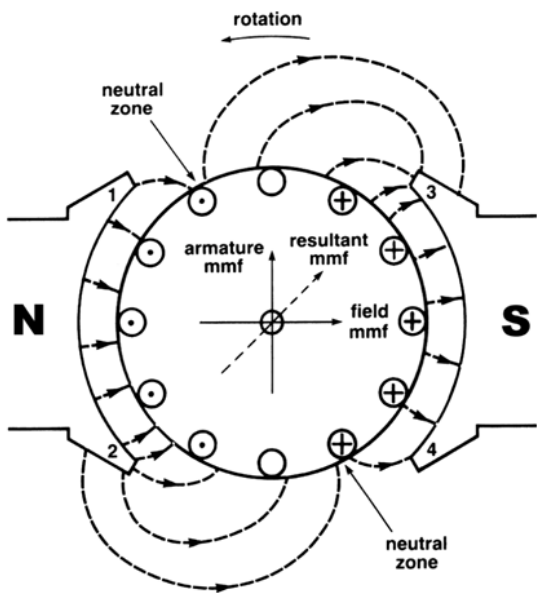


Figure 4.23: Dislocation of neutral zone

The second method, which is mostly used in practice, is to introduce *commutating poles*, as shown in *figure 4.24*. Commutating poles are electromagnets that are wound by a few turns of thick wires and placed in series with the armature winding and load. When current flows towards the load, it passes through the commutating poles that produce their own magnetic flux which is equal but opposite in direction to the armature flux. Hence, two fluxes cancel each other leaving only the original field flux and neutral zone is not disturbed. This method of countering armature reaction

does not depend upon load current as armature flux and commutating poles flux will always be equal and opposite, hence cancelling each other.

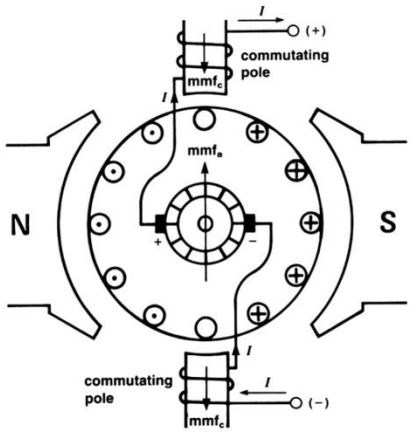


Figure 4.24: Commutating poles to tackle armature reaction

4.7 Types of DC Generators

Depending on their field, DC generators can be divided into three categories:

i. Separately excited generators
ii. Shunt generators
iii. Compound generators

4.7.1 Separately Excited Generators

In separately excited generators, field is either produced by a permanent magnet or electromagnet excited by a separate DC source. Hence, the field flux is constant and does not depend upon the induced voltage. *Figure 4.25* shows separately excited generator and its equivalent circuit diagram.

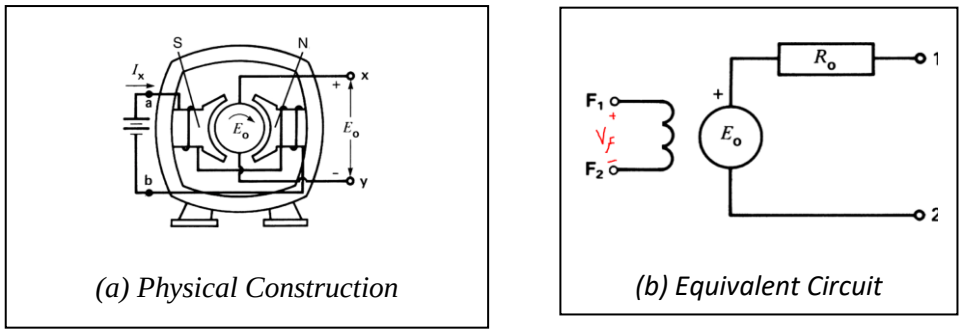


Figure 4.25: Separately Excited Generator

When field is excited by the field current, flux increases linearly. As current keeps on increasing, magnetic core of the field starts saturating and in turns field flux starts saturating. Usually, the current supplied to the field is such that the field flux just enters into saturation, as shown by the knee of the flux vs. current curve in *figure 4.26*. Remember, if the armature is turning at a constant speed, more voltage will be induced if field flux will be higher, i.e., if field current will be higher. If field flux is constant, turning the armature at a higher speed will induce more voltage as the rate of change of flux through armature coils will be higher.

To change the direction of induced voltage, direction of the field flux may be reversed, i.e., field current may be reversed by changing the polarity of field supply. Induced voltage direction may also be changed if armature is turned in the opposite direction. If both field supply polarity and direction of armature rotation is changed, induced voltage direction will not be changed.

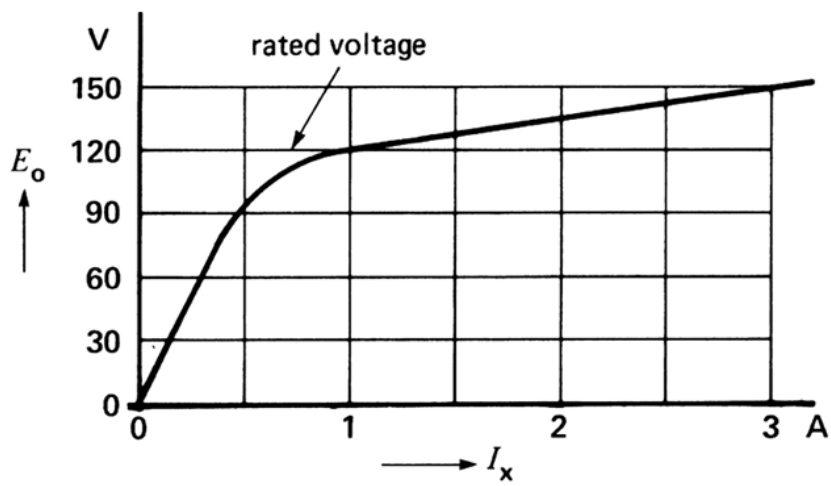


Figure 4.26: Induced voltage vs. field current

No-load terminal voltage for separately excited generators is the same as induced voltage. When a load is connected to the terminals and current starts flowing in the armature, terminal voltage drops due to some drop in the armature resistance. For separately excited generators, no-load to full-load terminal voltage drop is in the range of 5-10%

Example 4.1

A separately excited generator has lap-wound armature with 12 slots and 20 turns per coil. It has two poles with flux per pole to be 100mWb. Armature is turning at 100 rpm. If armature resistance is 1Ω and a 100 Ω load is connected to the terminals, calculate the terminal voltage.

Since armature is lap-wound, induced voltage may be calculated as follows:

$$Eo = \frac{Zn\phi}{60} = \frac{(12coils \times 20turns/coil \times 2conductors/turn)(100)(100\mathrm{E}-3)}{60} = 80V$$

Using voltage division, terminal voltage may be calculated as follows:

$$E_T = E_o \frac{R_L}{R_L + R_o} = 79.20V$$

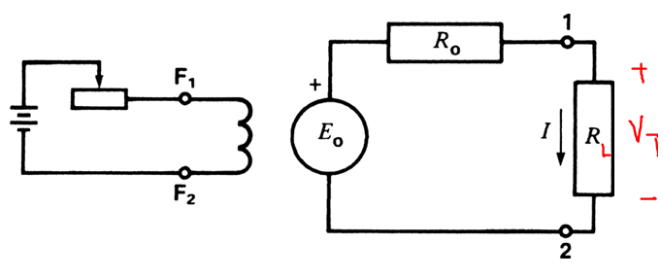
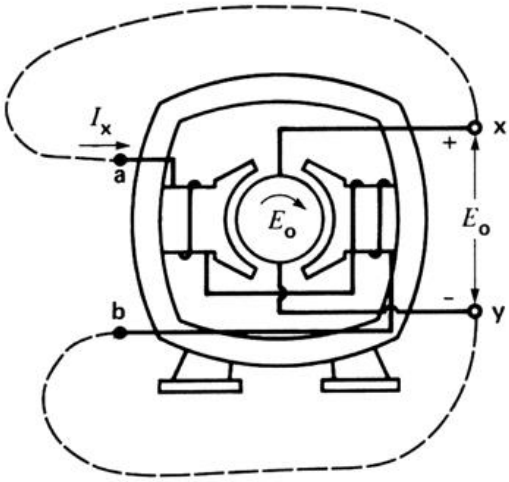


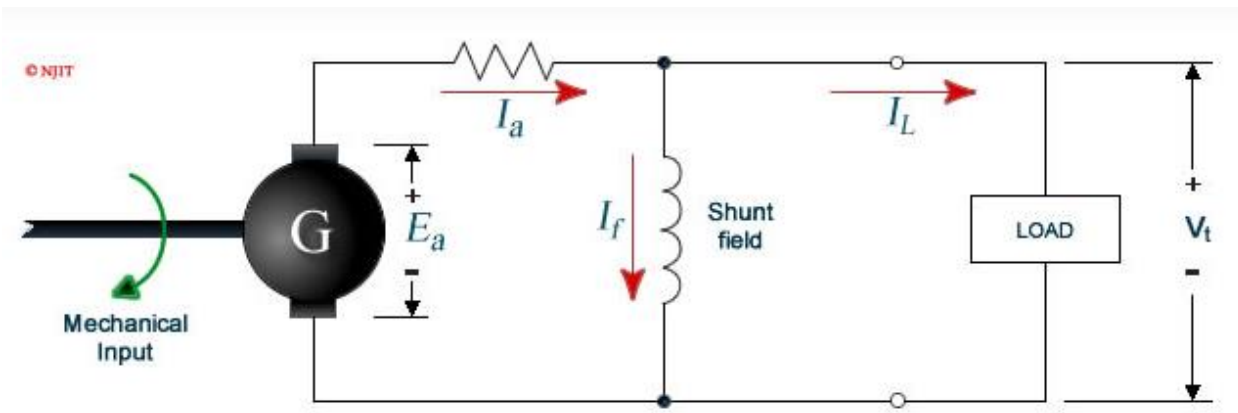
Figure 4.27: Separately excited generator under load

4.7.2 Shunt Generators

Shunt generators do not require any separate supply to excite their field to produce field flux. Field is connected in parallel or shunt with the armature winding. Recall from the discussion of hysteresis curve that once the magnetic material is magnetized by the current, removal of the current does not misalign all the magnetic poles and there is some residual flux always remain in the electromagnet. This residual flux provides the basis of setting up initial magnetic field that links to the armature. *Figure 4.28* shows a shunt generator and its equivalent circuit diagram.



(a)



(b)

Figure 4.28: (a) Shunt motor (b) Equivalent circuit

Note that in *figure 4.28*, equivalent circuit of a shunt motor has current I_a flowing through armature resistance, I_f flowing through field resistance, and I_L flowing through the load. Terminal voltage is applied across the shunt field which is equal to the induced voltage minus the drop in the armature resistance.

Working of a shunt generator can be explained as follows: When armature is rotated, it is linked with the residual flux from the field winding. This small residual flux induces a small voltage in the armature. This induced voltage runs a small current that goes to the field that increases the amount of field flux, which in turns induces more voltage in the armature. Now relatively higher

voltage produces more current that goes to the field and produces higher flux, which in turns produces even higher voltage. This cycle keeps on going until the induced voltage reaches to a maximum value determined by the field resistance and degree of saturation.

Controlling the voltage of a shunt generator:

A simple way to control the induced voltage of a shunt generator is to connect a rheostat (variable resistor) in series with the field winding that can be used to control the field current and in turn field flux that links to the armature coil, as shown in *figure 4.29*.

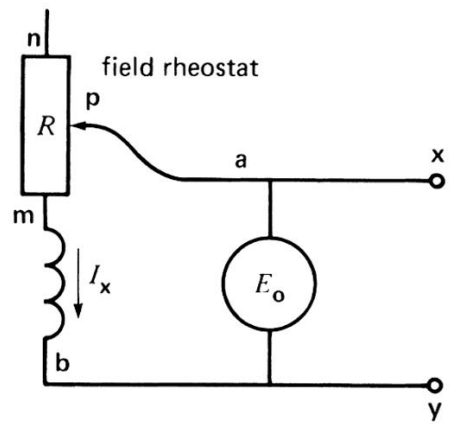


Figure 4.29: Rheostat field current control scheme for shunt generators

To control the field current, an initial value of rheostat is selected that produces flux to induce a base voltage. If induced voltage below the base value is required, rheostat value may be increased that will reduce the field current, hence, less flux will be produced that will lower down the value of the induced voltage. Likewise, if value above the base value is required, rheostat value will be reduced. This phenomenon is shown in the following graph between no-load voltage and field current.

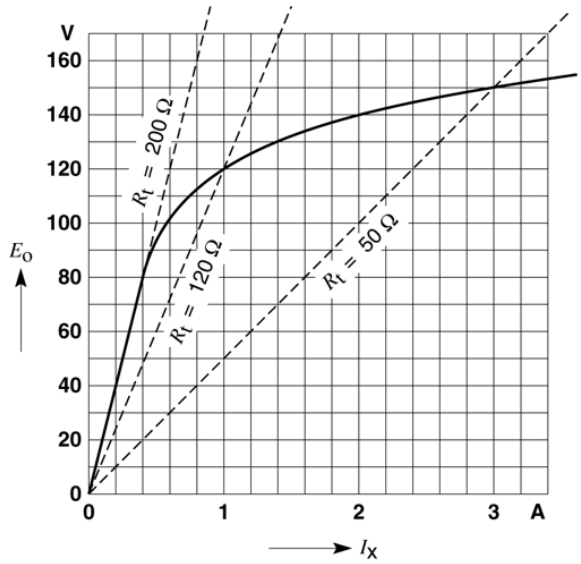


Figure 4.29: Terminal voltage at no load versus field current

Observe that the straight lines represent the relationship between induced or terminal voltage (neglect the small armature resistance) and field current using Ohm's law ($E_o = I_f(R_t+R_f)$) and curve represents the induced voltage per hysteresis phenomenon where increase in the field current after saturation point will not increase the flux appreciably. The intersection of two curves determines the value of the induced voltage or terminal voltage at no-load. Observe that if the rheostat value will keep on increasing to reduce the amount of field current to reduce the value of the induced voltage, at one point the straight line will become tangent to the hysteresis curve and will not intersect it. As soon as this will happen, induce voltage as well as the terminal voltage will go down to zero. Hence, there is a maximum value of the field rheostat beyond which it cannot be increased.

Effect of the load on the terminal voltage:

In separately-excited generators, induced voltage is not changed by the load. However, terminal voltage does change by the load as when current starts flowing in the armature, some voltage is dropped in it and terminal voltage becomes smaller than the induced voltage. In shunt generators, since field current also depends on the terminal voltage, as load increases and more current starts flowing out of armature, terminal voltage drops. This in turns reduces the field current ($If = E_T/R_f$), which in turns reduces the field flux and induced voltage further goes down. The no-load to full-load voltage in shunt generators is smaller than separately-excited generators. Generally, full-load voltage of shunt generators is about 15% less than the no-load voltage.

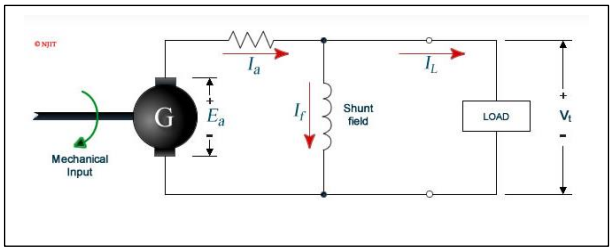
Example 4.2:

Induced voltage of a shunt generator is 100V, terminal voltage is 90V, field resistance is 20Ω, and armature resistance is 1Ω. Calculate armature current, field current and load current.

$$I_A = \frac{100-90}{1} = 10A$$

$$I_f = \frac{90}{20} = 4.5A$$

$$I_{load} = 10 - 4.5 = 5.5A$$



4.7.3 Compound Generators

Separately-excited generators require a separate source to excite the field. However, their terminal voltage under load is greater than shunt generators that do not require a separate source to excite their field. To have the benefit of self-excitation like shunt generators and higher terminal voltage, compound generators are designed. There are two different types of compound generators: *short*-shunt and *long*-shunt. In long-shunt compound generators, there is a field winding in series with the armature called *series field* and the combination is across the load and the shunt field. The other cumulative compound generator is *short-shunt*, where the series filed is in series with the load and armature is across the shunt field similar to shunt generators. The series winding is comprised of few turns of thick wires that produces flux when current flows to the load. Depending on the series field winding, compound generators can either be *cumulative* or *differential*. For our study, we will concentrate on short-shunt compound generators and refer to them as simply compound generators.

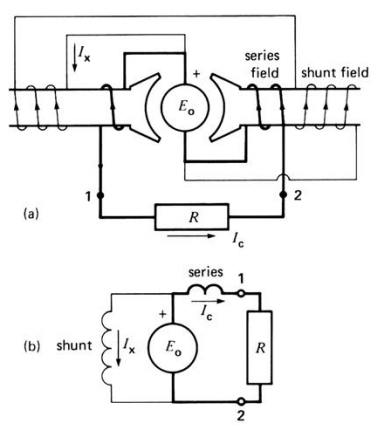


Figure 4.30: (a) Short-shunt Compound generator (b) Equivalent circuit

When load current flows in the cumulative compound generators, the series field flux is produced in the same direction as the shunt field flux, hence they are added together. This higher flux induces higher voltage in the armature which in turns produces higher terminal voltage. Hence, the terminal voltage either drops a little bit under full load or may not drop at all. In differential compound generators, series flux is in the direction opposite to the shunt flux, hence total flux goes further down and induced voltage is even smaller which in turns produces smaller terminal voltage. Due to this reason, differential generators do not find much practical use.

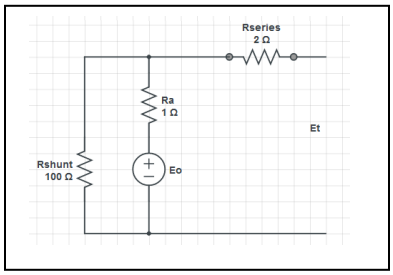
Example 4.3:

A cumulative compound generator has lap-wound armature with 100 slots, and 4 turns per coil. It is rotation with a speed of 100rpm. Flux per pole is 100mWb. Series field resistance is 2Ω and shunt field resistance is 100Ω, Armature resistance is 1Ω. Calculate:

(i) Induced Voltage:

$Z = slots*turns\_per\_coil*conductors\_per\_turn =$ 800 conductors

$Eo = Z*n*\phi/60 =$ 133.33V

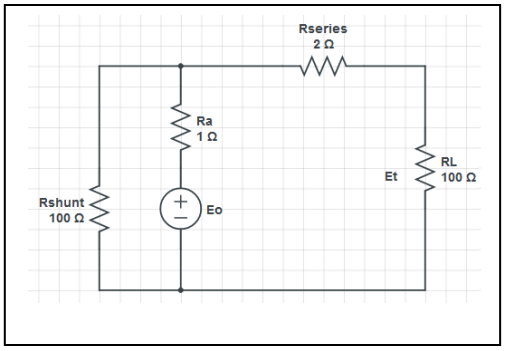


(ii) Terminal Voltage:

$Ia = Eo/(Ra + Rshunt) =$ 1.3201A

$Et = Eo - Ia*Ra =$ 132.01V

(iii) If a 100Ω load is connected to the terminals and it starts driving the current such that series field starts producing 10mWb flux per pole and shunt flux per pole drops down to 85mWb, calculate the load power.



$\phi_{new} =$ 95mWb

$Eo_{new} = Z*n*\phi_{new}/60 =$ 126.67V

$Req = (Rseries+RL)||Rshunt =$ 50.495Ω

$Ia_{new} = Eo_{new}/(Req+Ra) =$ 2.4598A

Using current division:

$IL = Ia_{new}*Req/(RL + Rseries) =$ 1.2177A

$P_{load} = IL^2*RL =$ 148.28W

4.8 Generator Specifications

Every generator has a nameplate with its *ratings* or *nominal characteristics*. These specifications may tell us about the rated power, rated terminal voltage, type of machine, shunt field current, speed of armature rotation for rated values, operable temperature, and insulation class. Load current as well as series field current (in case of compound generator) can be calculated by dividing the rated power by the terminal voltage. An example of generator nameplate is shown in *figure 4.31*.

Power	100KW	*Speed*	1200 rpm
Voltage	250V	*Type*	Compound
Exciting Current	20A	*Class*	B
Temperature Rise	50°C		

Figure 4.31: A DC Generator Sample Nameplate

The nameplate tells us that generator can deliver 100KW power at 250V, without exceeding the temperature rise of 50°C, when it rotates at the rated speed of 1200rpm. Hence, the maximum load current that it can supply is 100K/250 = 400A. It is a compound motor with shunt field current to be 20A. Series filed current depends on the load current but it cannot exceed 400A. Class *B* is the insulation type that corresponds to the materials that withstand temperature up to 130°C.

PROBLEMS

Note: Draw circuit for each problem.

1. A shunt generator has field resistor value Rf = 60Ω and no –load induced voltage rating Eo = 220V. A 60Ω resistor is added in series to the field winding to reduce field current by 60% and hence to reduce the maximum no-load generator induced voltage. What will be the new value of the induced no-load voltage? Ignore armature resistance. (*Note: Current is reduced by 60% that means the new current is 40% of the original value.)*

2. A compound generator has a lap-wound armature with 12 slots and 4 turns per coil. At no-load, flux per pole is 0.5Wb and it is running at 900 rpm. Shunt resistance is 100Ω, series field resistance is 3Ω, and armature resistance is 0.5Ω.
   (i) Calculate induced voltage, armature current and terminal voltage at no-load.
   (ii) If a 100Ω load is connected to the terminals and the series flux is added to the shunt flux such that it remains the same as it was at no-load, calculate the terminal voltage, armature current, and power dissipated in the load.

3. A separately-excited generator has lap-wound armature with 12 slots and 5 turns per coil. It is rotating at 300rpm. There are 6 poles and **total** flux linking the armature is 6 Wb. Armature resistance is 2Ω. There is a 100Ω load connected to the terminals. Calculate:
   (i) Induced voltage
   (ii) Armature current
   (iii) Terminal voltage
   (iv) Power delivered to the load

4. A shunt generator has an armature resistance of 1Ω and field resistance of 80Ω. Field coil has sixty turns and magnetomotive force there is 100 A.t.
   (i) Determine the induced voltage at no-load.
   (ii) Now, a load gets connected to the terminals that starts drawing 2A current at 210W. Calculate the new value of the induced voltage.
   (iii) What is the total input power?
   Note: Magnetomotive force is discussed in Chapter 2 under the topic of Magnetic Field Strength

5. A compound generator has 120V induced voltage. The armature resistance is 1Ω, shunt field resistance is 70Ω and series field resistance is 3Ω. Shunt field has 50 turns of thin wire and series field has 5 turns of thick wire.
   (i) Determine the armature current, terminal voltage, and *mmf* of the fields.

(ii) A load is connected that starts drawing 324W of power at a current such that the *mmf* of the series field is observed to be 15 A.t. Determine the terminal voltage, armature current, and the induced voltage.

6. A separately excited DC generator has no-load terminal voltage of 220V. The armature resistance is 1.5Ω. If a load of 50Ω is connected to the terminals, determine the terminal voltage, load current, and power consumed by the load.

7. A DC compound generator has a lap-wound armature with 200 slots and four turns per coil. It is rotating with a speed of 80 rpm and flux per pole is 100mWb. The armature resistance is 1Ω and shunt field resistance is 200Ω. The series field resistance is 2Ω. Determine:
   (i) Terminal voltage.
   (ii) If a load is connected to the terminals which starts pulling a load current such that the flux that is produced by the series field is added into the shunt field to keep the new induced voltage to be the same as the induced voltage at no load, what will be the new terminal voltage if the shunt field current is dropped to 90% of the shunt field current at no load?
   (iii) Power dissipated in the load.

8. A shunt DC generator has no load terminal voltage of 140V. Armature resistance is 1Ω and shunt field resistance is 50Ω. Calculate:
   (i) Induced voltage.
   (ii) A load is now connected to the terminals that starts drawing 2A current and the terminal voltage drops to 130V. Calculate the new value of the induced voltage.

9. A DC compound generator has no-load armature current of 1A. Armature resistance is 1Ω and shunt field resistance is 119Ω. Series field resistance is 4Ω. A 360W load gets connected to the terminals that starts drawing load current of 4A. Determine the induced voltage at no-load and full-load.

Chapter 5–Direct-Current Motors

5.1 Principle of Operation

Construction of DC motors is similar to DC generators except load is replaced by a DC power supply that supplies current to the armature. When this current is interacted with the field flux, Lorentz force is acted upon the armature (rotor), which applies a torque on it, and it starts rotating about its axis. *Figure 5.1* shows a basic DC motor.

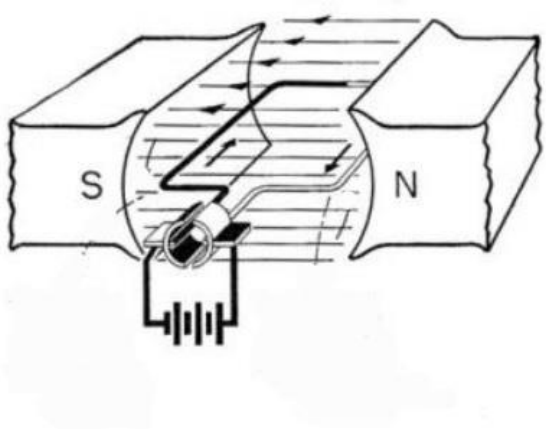


Figure 5.1: A simple single loop DC motor

Observe that in *figure 5.1*, magnetic field is from right to left (**-j**). Since current is going into the page (**-i**) under S-pole, force on that conductor will be in the direction of **L×B,** i.e., **-i×-j = k,** i.e., upward. Likewise, the conductor under N-pole will experience a downward force. Therefore, direction of rotation of the armature will be clockwise.

DC motors provide a very large starting torque, as will be discussed soon, and in general their speed-torque characteristics can be varied over a wide range while maintaining high efficiency; that's why electric cars do not have any gears.

5.2 Counter-electromagnetic Force (CEMF)

As initial current flows in the armature and it is acted upon by the Lorentz force and armature starts rotating, magnetic field interacting with the armature starts changing. This changing

magnetic field induces a voltage in the armature coils according to Faraday's law of electromagnetic induction.

Observe from *figure 5.1* that if the coil is rotating clockwise, the voltage induced in the conductor **(v×B)** will **k×-j = i**, i.e., along positive *x*-axis under S-pole, and along negative *x*-axis under N-pole. Therefore, the induced voltage is in the direction opposite to the applied voltage, therefore, it is called counter-electromotive force. Circuit diagram of a DC motor is shown in *figure 5.2*.

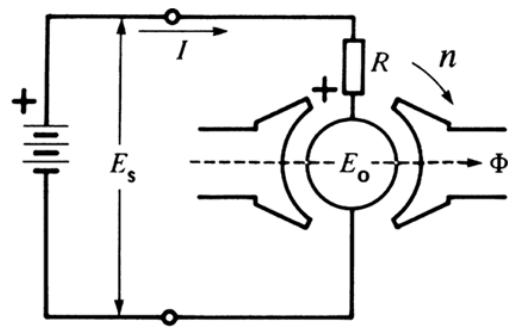


Figure 5.2: Circuit diagram of a DC motor. 'R' is the armature resistance and 'E_o' is cemf

If the armature is lap-wound, the value of the counter emf may be calculated using (4.1) given previously for DC generators:

$$E_o = \frac{Zn\phi}{60} \tag{5.1}$$

<u>*Example 5.1*</u>

A lap-wound motor with 10 coils, 2 turns per coil is rotating at 100rpm. Flux per pole is 3Wb. Armature resistance is 1Ω and power drop in the armature is 10W. Calculate induced voltage (cemf) and supply voltage.

Refer to *figure 5.2:*

Induced EMF Eo = (10×2×2×100×3)/60 = 200V

Armature Current $I = \sqrt{(P/R)}$ = 3.16A

$\therefore E_s = IR + E_o$ = 3.16 + 200 = 203.16V

5.3 Acceleration of the Motor

When a motor is running at its nominal speed at no load, the value of the *cemf* developed is quite close to the source voltage and the current flowing in the armature may be measured using Ohm's law:

$$I = \frac{E_s - E_o}{R} \tag{5.2}$$

This current is quite small as difference between the source and induced voltage is small. When motor starts from stand still, there is no voltage induced in the armature, hence, the armature current is simply given by,

$$I = \frac{E_s}{R} \tag{5.3}$$

where R is the total resistance in the path of the armature current.

This current is very large as compared to the no-load rated current (about 20-30 times large). This large current puts a large force on the armature, which in turns deliver a large torque to move the motor from stand still. Once armature starts rotating, voltage starts inducing in it. This reduces the amount of current in the armature according to (5.2). As current goes down, force on the armature goes down and acceleration becomes slower. The armature will keep on accelerating until it will reach to the point where induced voltage becomes very close to the supply voltage (at no load). After that the armature will start cruising at a constant speed, which is the rated motor speed at no-load. Note that the small current required at this point goes towards the compensation of small losses and small force required to keep the armature turning with a constant velocity.

Note that the induced voltage will never be equal to the source voltage. Theoretically, if it does happen then the current in the armature will become zero, which in turns will make the Lorentz force zero and the armature will start slowing down. As soon as it slows down, induced voltage goes down and current again starts flowing in the armature. This, again, puts a force on the armature that applies a torque and it will accelerate again. Hence, at no-load, the source voltage is always slightly larger than the induced voltage to have a small current flowing in the armature to produce a small torque required to keep it moving at a constant speed.

Example 5.2

A DC motor is connected to a 150V supply and has an armature resistance of 1Ω. Calculate the starting current. If the induced voltage developed at 2000rpm is 80V, calculate the induced voltage and the armature current if motor is running at 3000rpm.

Starting current = Es/R_A = 150A

Since $E_o = \frac{Zn\phi}{60} = kn \Rightarrow k = \frac{E_o}{n}$ =80/2000 = 0.04, $\therefore$ at n = 3000 rpm, $E_o = kn$ = 120V

and Armature current $I = \frac{E_s - E_o}{R}$ = (150-120)/1 = 30A

5.4 Mechanical Power & Torque

In a separately-excited motor, power supplied to the armature can be calculated by,

$$P_A = E_s I_s = (E_o + I_s R_A) I_s = E_o I_s + I_s^2 R_A \tag{5.4}$$

Note that $E_o I_s$ is the electrical power converted into mechanical power by the armature and $I_s^2 R_A$ is the power dissipated as heat in the armature resistance.

The relationship between power and torque may be given by,

$$P = \frac{nT}{9.55}$$
$$\Rightarrow T = \frac{9.55P}{n} = \frac{9.55E_o I_s}{n} = \frac{9.55Zn\phi I_s}{60n} = \frac{Z\phi I_s}{2\pi} \tag{5.5}$$

since for lap-wound motors $E_o = \frac{Zn\phi}{60}$.

5.5 Speed of Rotation

When a motor is running at the rated speed, counter electromotive force (E_o) is quite close to the supply voltage *(Es)* since drop in the armature resistance is small.

$$\therefore Es \approx Zn\phi/60 \tag{5.6}$$

Since total number of conductors Z is constant, speed depends upon the source voltage and flux per pole. From (5.6), one can deduce,

$$n \propto E_s \text{ (if } \phi \text{ is constant) or}$$
$$n \propto \frac{1}{\phi} \text{(if } E_s \text{ is constant)} \tag{5.7}$$

This shows that speed of rotation increases with the increase in the source voltage, as long as flux is constant, and speed of rotation decreases with the increase of flux per pole, if source voltage is not changing.

5.6 Types of DC Motor

There are various types of DC motors as far as configuration of field and armature is concerned. These are *separately-excited, shunt, series,* and *compound* motors. These motors are explained in the following sub-sections.

5.6.1 Separately-Excited Motors

Separately-excited motors have their field set-up by an electromagnet which is excited by a separate DC supply as shown in *figure 5.3*.

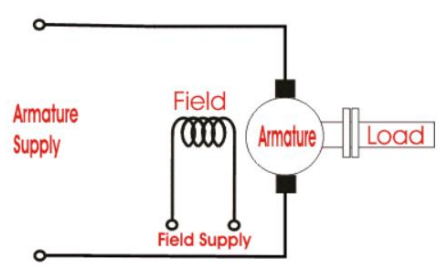


Figure 5.3: Separately-excited DC motor

In this case as long as the field supply is not changing, flux linked with the armature will not change and speed of motor will only depend on the armature supply voltage. Although, armature

resistance is not explicitly shown in *figure 5.3* but it is always present. In this figure, assume that it is part of the *armature* block.

To control the speed of a separately-excited generator, a special arrangement may be used, which is called *Ward-Leonard Speed Control System*. This is a very practical system that has various applications. The system is shown in *figure 5.4*.

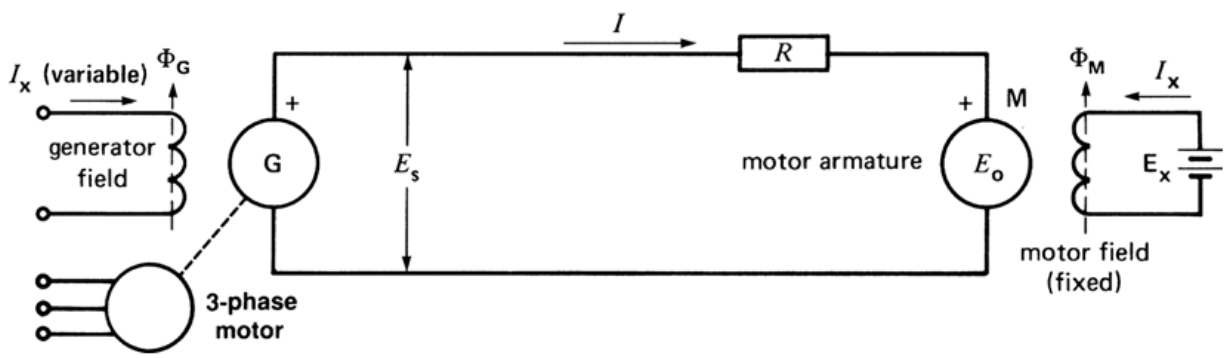


Figure 5.4: Ward-Leonard speed control system

In this system, supply voltage is provided by a DC generator whose armature is rotated by a 3-phase motor. Hence, the supply voltage can be changed by either changing the generator field flux or by changing the speed of armature rotation. Also, if direction of *Es* is made reversed by either changing the direction of rotation of the 3-phase motor or by reversing the field supply, it will also reverse the direction of rotation of the motor. In general, *Es* can be changed from zero to a maximum value, which can change (increase) the motor speed from zero to a maximum value. If motor is needed to be rotated in the opposite direction, polarity of *Es* is reversed. This system has various applications including, but not limited to, elevators, steel mills, mines, paper mills etc.

If $E_s > E_o$, positive torque will develop in the motor and power will flow from the generator to the motor, as shown in *figure 5.4*. If *Es* is reduced and E_o becomes larger than *Es*, motor will start acting like a generator and start sending current back to the generator *G*. This will reverse the torque on the motor, and it will start slowing down; hence, *electrical* brakes are applied to the motor. The power received by the generator from the motor is fed back into the AC line that usually feeds the AC motor. Hence, power can be recovered in this way, and this is a big advantage of Ward-Leonard system. This recovery of power is along the same lines as *regenerative braking* systems, where power is recovered during braking of a system that is fed back to the system.

Example 5.3

A 10KW, 600V lap-wound motor is driven by a 12KW generator using a Ward-Leonard system. Total motor and generator armature resistance is 0.5Ω. The nominal speed of the motor is 350rpm when the induced voltage 580V. Calculate the motor torque and speed when Es = 450V and Eo = 410V.

Refer to *figure 5.4*, since motor is lap wound, $Eo = Zn\phi/60 \rightarrow Eo = kn$, where $k = Z\phi/60$,

$\rightarrow k = Eo/n = 580/350 = 1.6571$.

Therefore, when $Eo = 410V$, $n = Eo/k = 410/1.6571 = 247.41rpm$.

The armature current when $Es = 450V$ and $Eo = 410V$: $Ia = (Es-Eo)/Ra = 80A$

Output power of the motor: $Po = E_oI_a = 32.8KW$

Output torque: $T = 9.55*Po/n = 1266.1Nm$

5.6.2 Shunt Motors

Shunt motors have their field connected to the same supply that provides current to the armature; hence, armature, source, and field all are parallel to each other, as shown in *figure 5.5*.

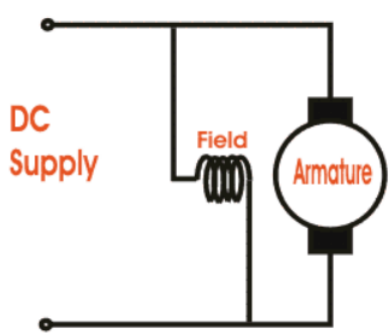


Figure 5.5: Shunt Motor

Once again, armature also has a resistor that is not shown in *figure 5.5*.

There are two ways to control the speed of a shunt motor; *rheostat (armature) speed control* and *field speed control*. Rheostat speed control employs a rheostat (variable resistor) in series with the armature, as shown in *figure 5.6.*

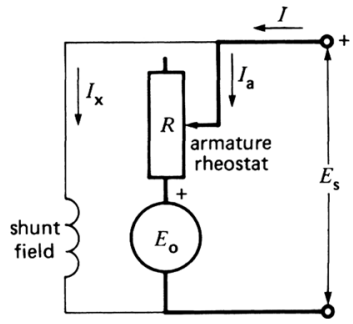


Figure 5.6: Rheostat speed control

When the value of this rheostat will increase, current in the armature will decrease. This will produce smaller Lorentz force on the armature which in turns will reduce the torque and speed of the armature. This will also cause the induced voltage (cemf) to go down. The downside of this speed control method is that it will produce a lot of heat in the armature rheostat, especially for large motors that require large currents to move heavy loads.

The other method is to introduce this rheostat in the field instead of the armature, as shown in *figure 5.7*

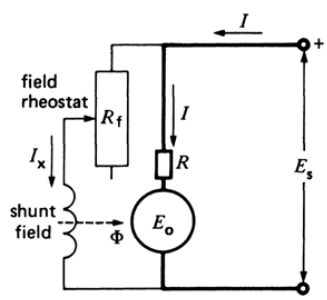


Figure 5.7: Field speed control

When field rheostat is increased, field current will decrease. This consequently will reduce the flux linking to the armature. When flux goes down, armature speed increases. This method is used when a motor has to run above the rated speed, called *base speed*. Complete explanation of the

process is as follows. When the field resistance is increased, field current and flux are reduced. This in turns will induce less voltage in the armature (*Eo)*. Since induce voltage goes down, armature current increases ($I_A = \frac{E_s - E_o}{R_A}$). This in turns will increase the Lorentz force on the armature ($\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$) and with higher torque, the armature will speed up. With the increase of the speed, armature will now be cutting the weak field faster; hence, rate of change of flux cutting the armature will increase and induce voltage will go up again. Therefore, the new induced voltage will be very close to the old one with weaker flux and higher speed ($E_o = \frac{Zn\phi}{60}$). If flux becomes very weak, an extremely large speed will be required to have almost constant induced voltage. This situation may put a machine at risk of the mechanical breakdown, and it is called *runaway* condition. Safety devices must be placed to prevent flux to go down too low that may result in the runaway condition.

Effect of Load

When a load is connected to a shunt motor running at the rated speed without any load, it slows down as the torque required by the load is larger than the no-load torque of the motor. As soon as it slows down, induced voltage goes down ($E_o \propto n$). Hence, armature current increases. With the increase in the armature current, armature will experience higher Lorentz force which in turns will produce more torque and motor will accelerate again. It will accelerate until the motor torque becomes equal to the load torque. At that point it will no longer accelerate and will start rotating with a constant speed. For shunt motors, no-load to full-load speed is 10-15% different. Note that if speed of the motor is needed to be increased with load connected, field current can be reduced with a field rheostat that will reduce the flux and increase the speed. Hence, an adaptive motor control can be designed that can automatically change its field flux based on the load connected and run the shaft at a constant speed.

Starting a Shunt Motor

When a DC motor is started, an extremely large current flows in the armature since counter EMF is not developed yet. This current is usually ten to twenty times larger than the full-load current. If not controlled properly, this large starting current may damage armature wiring and insulation and even snap-off the shaft due to sudden mechanical shock. Hence, special consideration must be taken into account to limit this starting current. A general idea is to connect a rheostat in series with the armature whose value gradually decreases as motor speeds up. One special technique to control the starting current of shunt motors is through *face-plate starters*, as shown in *figure 5.9*

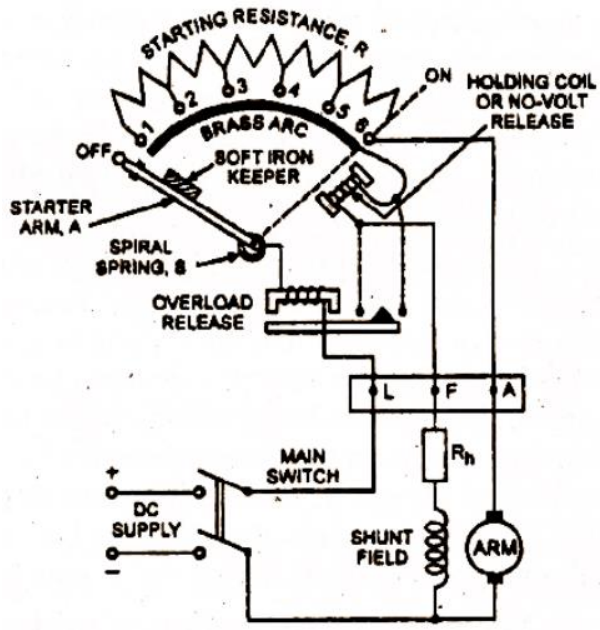


Figure 5.9: Manual Face-Plate Starter

Initially, the starter arm is in the OFF position and no current is flowing in the field and the armature. When the motor is switched ON, the starter arm moves towards right and connect to the contact 1 as well as the brass arc. The brass arc provides full current to the field and armature starting current is limited through the series of starting resistors. When the motor starts moving and attains a constant speed, starter arm is moved to connector *2*, thus reducing the total series resistance in the path of the armature. The armature current increases, which also increases the motor speed. This process keeps on going until the starter arm moves all the way to the connector *6*. At this time, motor starts moving at its rated speed and rated armature current starts flowing in the armature. Observe that there are two electromagnets, *no volt release* and *overload release*. When the starter arm is all the way to the right, it is held in its position by the magnetic attraction provided by *no* volt release electromagnet. If for some reason input supply voltage gets interrupted or field current gets interrupted, this electromagnet discharges and releases the starter arm which is pulled towards left to the OFF position through the spiral spring. Same thing happens if the motor becomes overloaded. This is a safety precaution so that a dip in the field current and field flux would not momentarily increase the speed of the motor. Also, an interruption in power supply will lead to flow of a large armature starting current if the starter arm is left at connector *6* when the power turns back on after interruption.

Example 5.4

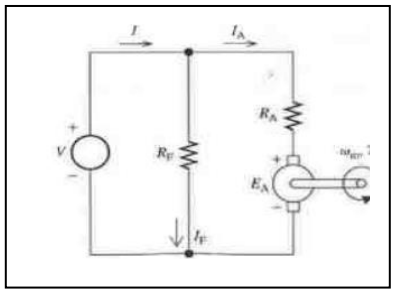
A shunt motor is connected to a 120V source, which is supplying 51A total current to the motor. The shunt field resistance is 120Ω and armature resistance is 0.1Ω, Calculate:

Armature current: If = V/Rf = 1A

→*Ia = I-If =* 50A

CEMF: $E_a = V - I_aR_a =$ 115V

Mechanical Power of the motor: $Pa = E_aI_a =$ 5.75KW



5.6.3 Series Motors

Instead of a shunt field, a few turns of thick wiring is wound on electromagnet which is placed in series with the armature and input source *Es*, as shown in *figure 5.8*

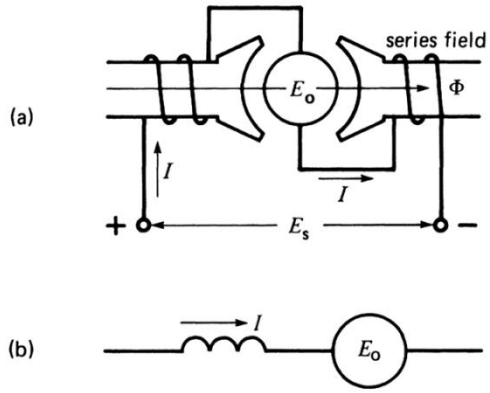


Figure 5.8: (a) Series motor construction (b) Circuit diagram

The field flux in series motors changes as armature current changes, unlike shunt motors where flux is constant. When series motor starts, a large current flows through the armature (field coil has extremely low resistance and armature resistance is low as well) since E_o is zero. This large current produces a large flux which in turns applies a large Lorentz force on the armature. This results in an extremely large starting torque (larger than shunt motors) capable of moving quite heavy loads from rest. When a series motor is operating at full load and counter EMF is not close to the source voltage, a good amount of current flows through the armature and the flux per pole

is approximately equal to that of a shunt motor with identical power and speed. On the other hand, if motor is not operating at full-load and induced voltage is close to the source voltage, armature current is small which results in a smaller flux, which leads to an increase in the motor speed. This may result in the runaway condition. Therefore, series motors are never operated at no load.

Series motors are used where high initial torque is required. They are also used when light loads are to be moved at high speeds. They are used for traction purposes and also in electric cranes and hoists. Power of series motors is pretty constant at all speeds as there is more torque at lower speeds and vice versa ($P = \frac{nT}{9.55}$)

Speed Control

There are two ways to control speed of series motors: If higher than the base speed is required, place a rheostat in parallel to the series field winding. This will split the current going in the field and hence will reduce the flux which in turns will increase the speed. If less than the base speed is required, a rheostat can be connected in series with the armature. This will result in a drop of voltage in the rheostat and field winding and less voltage will be available for the armature. This will result in a reduction of speed.

5.6.4 Compound Motors

Like compound generators, compound motors have both shunt and series windings, as shown in *figure 5.10*

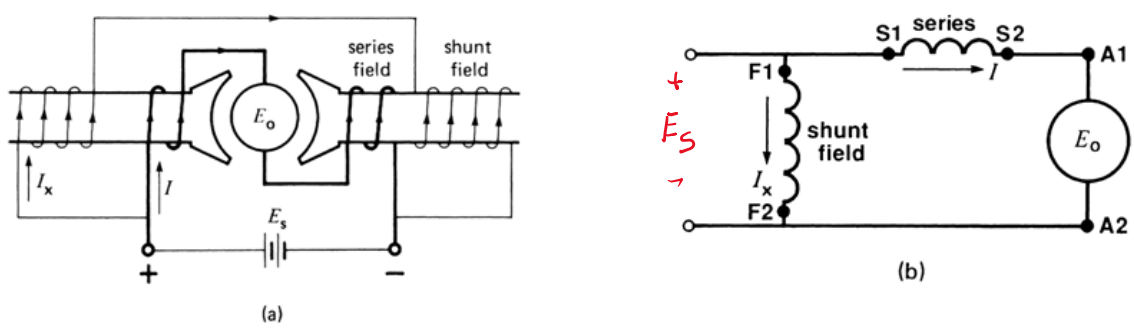


Figure 5.10: (a) DC Compound Motor (b) Circuit diagram without armature resistance

The series field winding is comprised of a few turns of thick wires with very low resistance. There are two types of compound motors; *cumulative* and *differential*. Cumulative compound motor field

is the combination of both shunt and series fields whereas differential motor field is the difference of both fields.

At no load and rated speed, the motor acts as a shunt motor since armature current is very small and series field is negligible. As motor is loaded, it slows down, which decreases *Eo* and armature current increases. This increases the series field flux which adds up with the shunt field flux for the cumulative motors. Since for lap-wound motors, speed is inversely proportional to the flux per pole, increase in flux further reduces the speed of rotation. If a motor is differential, as load increases and series flux increases, total flux goes down. This results in an increase of speed. This phenomenon may lead to instability. Due to this reason differential compound motors do not find many practical uses.

The speed drop from no-load to full-load for cumulative compound motors is from 10%-30%. *Figure 5.11* shows the speed-torque relationship for different motors.

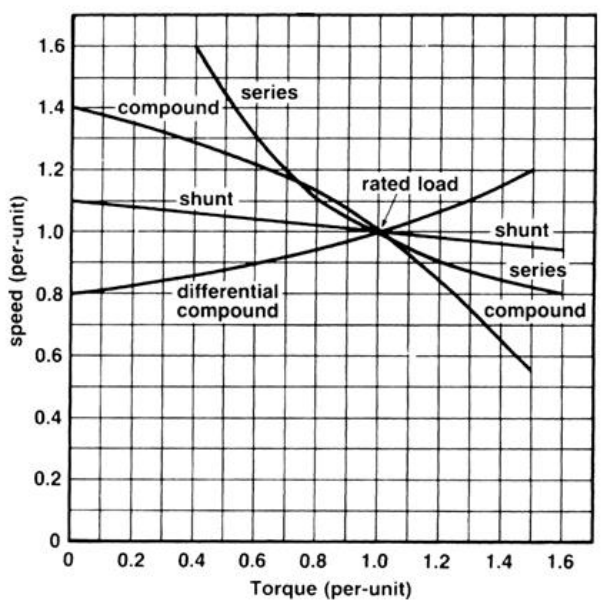


Figure 5.11: Speed-Torque relationship for different motors

Observe that as torque requirement increases (motor is loaded), speed decreases for all the motors except differential compound motors which acts in the opposite way.

To change the direction of rotation of compound motors, either armature connection with the source can be reversed or both shunt and series field connections can be reversed.

5.7 Stopping A DC Motor

When a motor is loaded and running at its rated speed, simply by turning the power off may not make the motor stop quickly as it may be carrying a large weight whose inertia may take a long time before motor finally stops. Take the example of an automobile; if you are cruising at 60 mph and then you just let the accelerator go without applying brakes, inertia of the car will move it for a good long distance before finally it will stop due to frictional forces. Hence, to stop a loaded motor quickly two techniques may be used: *mechanical braking* and/or *electrical braking*.

Mechanical braking is simply the application of mechanical brakes to the shaft of the motor to make it stop. Electrical braking, which is used very commonly, is to change the direction of the current flow to the armature to apply opposite Lorentz force, and hence, opposite torque to the direction of rotation to make the motor stop quickly. There are two types of electrical braking methods: *dynamic braking* and *plugging.*

Dynamic Braking

In dynamic braking, when motor needs to be stopped, the armature is disconnected from the supply and connected to a *braking* resistor, as shown in *figure 5.12*

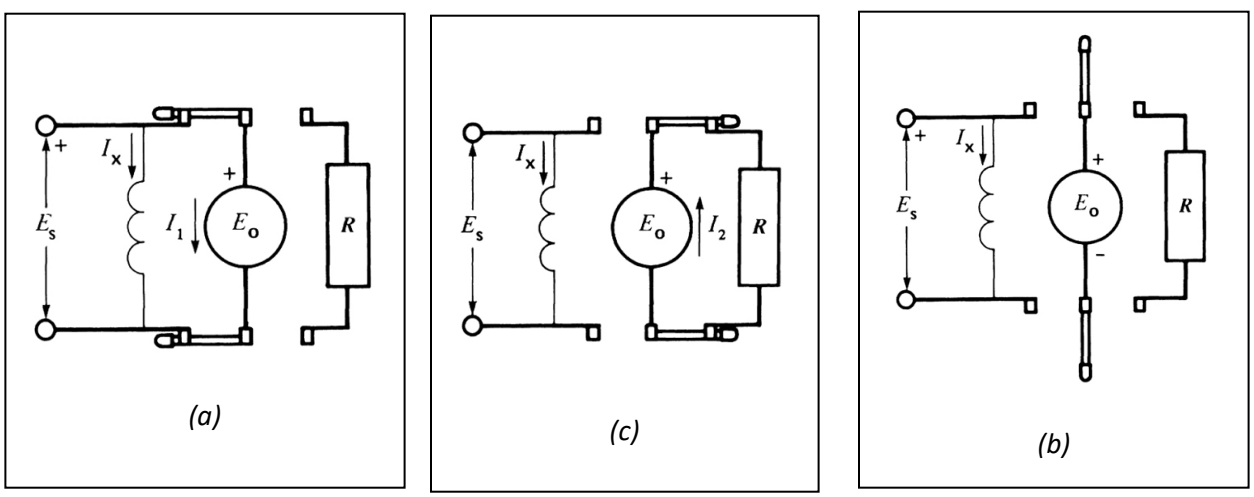


Figure 5.12: (a) Armature connected to supply (b) Armature disconnected from supply (c) Armature connected to the braking resistor

Generally, braking resistor value is chosen such that the initial braking torque is two to three times more than the load torque. As the braking resistor is connected to the armature as shown in *figure*

5.12(c), it starts supplying current to it. Since the direction of current is opposite to the one flowing in the armature when it was connected to the supply (*figure 5.12(a)*), opposite Lorentz force is applied on the armature which applies a torque opposite to the direction of rotation and it starts slowing motor down. As mentioned earlier, braking resistor value is chosen such that the initial braking torque is quite large, which starts slowing motor down quickly. As motor slows down, the induced voltage value goes down exponentially and speed of the motor also goes down exponentially until it stops.

Plugging

Plugging is a technique where instead of disconnecting the armature from the power supply and connecting it to a braking resistor, the supply connected to the armature switches its polarity. Hence, armature current changes its direction, applying a negative torque on it which slows it down. Arrangement of this technique is shown in *figure 5.13*

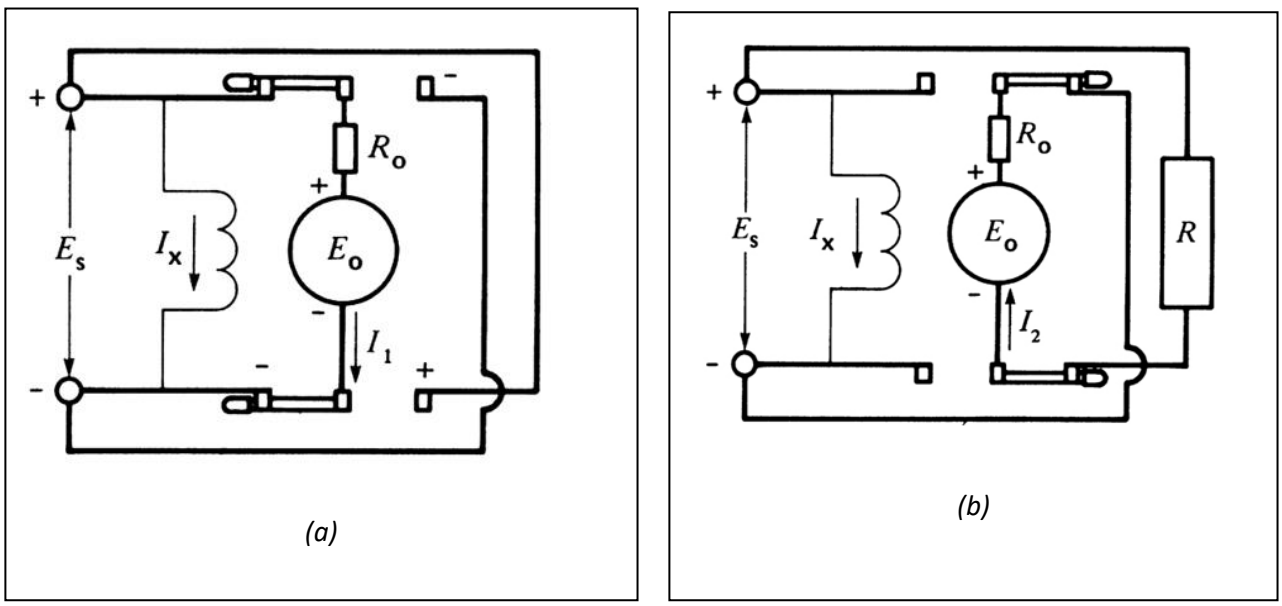


Figure 5.13: (a) Armature connected to the supply (b) Supply switches its polarity

When supply switches its polarity and current direction through armature is reversed, direction of the Lorentz force on the armature is reversed as well, which applies an opposite torque on the armature and it starts slowing down. Care should be taken to disconnect the supply as soon as motor will stop otherwise it will start rotating in the opposite direction. *Figure 5.14* shows speed

versus time curves for both the techniques compared against motor slowing down on its own due to friction.

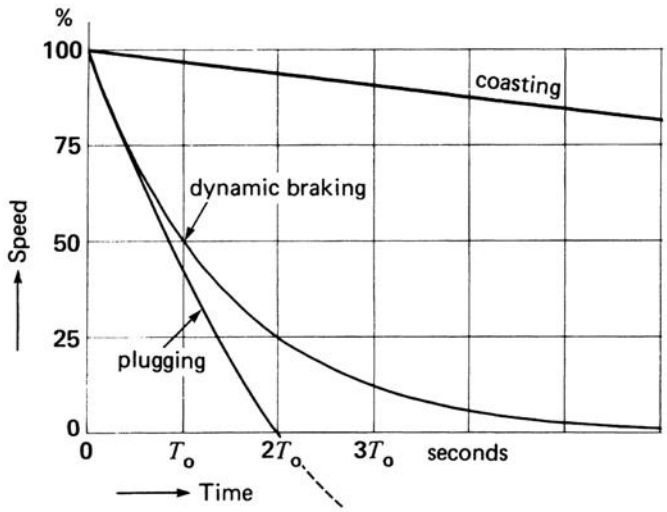


Figure 5.14: Speed vs. Stopping Time

<u>Example 5.5</u>

A 120hp lap-wound shunt motor is connected to a 240V source and rotating at 400rpm. Nominal armature current is 400A. Dynamic braking is used to stop the motor. Calculate:

(i) Value of the braking resistance if maximum braking current is limited to 125% of its nominal current value. Ignore the armature resistance.

Refer to the circuit shown in *figure 5.13*, the maximum braking current will flow as soon as the dynamic braking will be applied. Since armature resistance is ignored, therefore, $Eo \approx Es$, and braking resistance $R_B = Es/I_B(max) = Es/(1.25*400) =$ 0.48000Ω

(ii) Braking power if motor has decelerated to 150rpm.

$Eo = Zn\phi/60 \rightarrow Eo = kn$, where $k = Z\phi/60$, $\rightarrow k = Eo/n = 240/400 = 0.6$

$\rightarrow Eo$ at 150rpm $= kn = 90$V

Current in the braking circuit: $I_B = Eo/R_B = 187.5$A

Therefore, braking power $= E_oI_B =$ 16.875KW

Note: If the armature resistance is given, you will have to consider it in the braking circuit, which will be in series with the braking resistance.

5.8 Armature Reaction

Like in generators, when substantial amount of current flows through armature in motors under load, it produces its own magnetic field. This magnetic field interacts with the magnetic field produced by the field electromagnets and results in a net field that not only changes the pattern with which total flux links with the armature coils and commutator sections, thus producing sparks at the brushes by shifting the neutral zone, but may also speed up the motor by weakening the total flux. The problem of armature reaction in motors may be handled the same way as in DC generators by using *commutating poles*.

5.9 Motor Efficiency

The efficiency of a motor is the ratio of its output mechanical power to the input electrical power. It is generally expressed in percentage,

$$\eta = \frac{P_o}{P_i} \times 100\% \tag{5.8}$$

The difference between the input and the output power is account for losses in the motor. The smaller the losses, the higher the efficiency and vice versa.

In this chapter, losses account for *conductor* or *copper losses*, which are losses in the armature and field windings. These losses can easily be calculated using the power formula I^2R or simply the difference between the input and output power.

PROBLEMS

Note: Draw circuit for each problem

1. A shunt motor is connected to 230V line and running at 1000r/min. Armature current is 60A and total armature resistance is 0.15Ω. Field resistance is 50Ω. Armature is lap-wound with 83 slots and 6 turns per coil. Calculate,
   (i) CEMF
   (ii) Conductor loss
   (iii) Mechanical power of the motor in KW and HP
   (iv) Torque developed by the motor
   (v) Flux per pole
   (vi) Efficiency of the motor

2. A shunt motor has armature resistance of 1Ω. It is connected to 110V line and rotating at 3000 r/min. CEMF is 90V and armature is lap-wound. Calculate,
   (i) Starting current
   (ii) Armature current at the given induced voltage.
   (iii) If dynamic braking is used and initial braking current required is 1.5 times the armature current in (ii), determine the value of the braking resistor.
   (iv) CEMF and the armature current when motor runs at 2000 r/min

3. A shunt motor is connected to line with 110V and rotating with 2000 r/min. Line current is 40A. Field resistance of motor is 100Ω and armature resistance is 0.2Ω. Calculate,
   (i) Armature current
   (ii) CEMF
   (iii) Mechanical Power

4. A compound motor has 1200 turns on the shunt winding and 25 turns on the series winding, per pole. The shunt field has a total resistance of 115Ω, and the armature current is 23A (at full load). If the motor is connected to a 230V line, calculate,
   (i) The magnetomotive force, *mmf*, per pole at full load
   (ii) The magnetomotive force, *mmf*, at no-load

5. A 300hp, 240V, 600 r/min motor is required to be stopped with dynamic braking technique. If the nominal armature current is 100A, calculate,
   (i) The value of braking resistor *R* if we want to limit the maximum braking current to 150% of its nominal value
   (ii) The braking power (kW) when motor has decelerated to 200 r/min

   Assume armature resistance to be negligible for this problem.

6. A separately-excited DC motor has lap-wound armature with 100 slots and 2 turns per coil. It is rotating at 120 rpm and flux per pole is 0.1wb. Armature resistance is 1Ω and armature loss is 10W. Determine:
   (i) Source voltage
   (ii) Starting armature current

7. A shunt DC motor is connected to a load which is pulling 3A armature current. Armature resistance is 1Ω. The load requires an output power of 120W and efficiency of the motor is 82%, calculate:
   (i) Source Voltage
   (ii) Starting armature current
   (iii) Shunt resistance and shunt current

8. A separately excited motor has negligible armature resistance. It is connected to a 100V source voltage and running at 1000 rpm. Flux per pole is 1Wb. There is a sudden interruption to the field voltage such that the flux is increased to 1.3Wb. How it is going to affect the speed of the motor and what will be the new speed? Armature is lap-wound.

9. A shunt motor has lap-wound armature with 100 slots with 2 turns per coil. It is rotating at 100rpm. Flux per pole is 100mWb. Armature resistance is 1Ω and field resistance is 90Ω. Output mechanical power is 200W. Determine the efficiency of the motor.

10. A DC shunt motor is connected to a 220V source. Shunt field resistance is 150Ω. Armature resistance is 1Ω. Determine:
    (i) CEMF if the full-load current is $1/20^{th}$ of the starting armature current.
    (ii) Motor efficiency (assume there are no friction and iron losses).

(iii) If dynamic braking is used to stop the motor, determine the value of the braking resistor if the initial braking current is three times the full-load armature current.

(iv) Determine the value of CEMF and braking power when braking current drops down to the same value as the full-load armature current.

Chapter 6 - Efficiency & Heating of Machines

6.1 Types of Power Losses

There are two types of power losses in machines: *mechanical* and *electrical.*

Mechanical Losses

There are two categories of mechanical losses: *frictional losses* and *windage losses*. *Frictional* losses come from any moving part in a machine including, but not limited to, brushes, bearings, commutators, and slip rings. *Windage* losses come from the air friction produced by cooling fans to remove heat from a machine.

Electrical Losses

There are three different categories of electrical losses: *conductor losses, brush losses,* and *iron losses*.

(i) Conductor Losses: Heating loss when current flows in the field and armature conductors.

(ii) Brush Losses: There is always some voltage drop at the brush and commutator or brush and slip ring contact. If a brush gets old or if there is improper contact, there is more voltage drop at the contact.

(iii) Iron Losses: This loss is due to the hysteresis phenomenon in the core as well as due to heating of the core because of eddy currents. Iron losses impose a mechanical drag on the armature and their effect is like producing mechanical friction. This is generally measured in terms of torque calculation due to the iron loss.

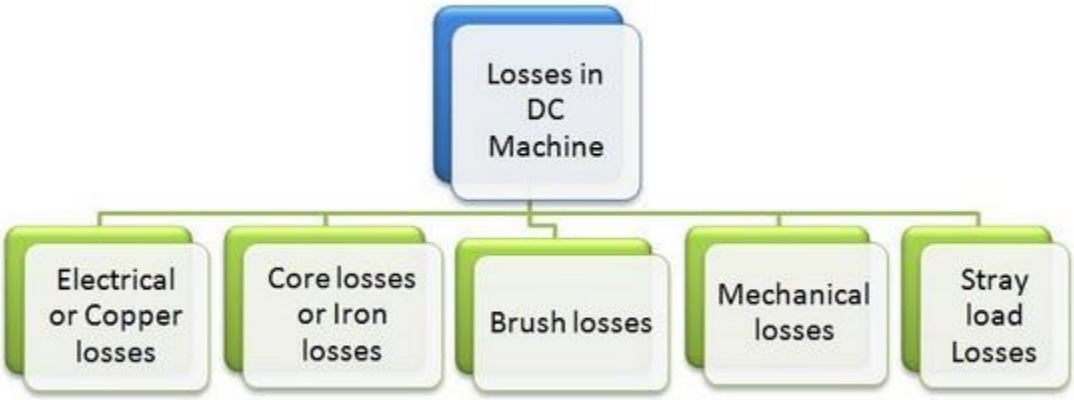


Figure 6.1: Losses in a DC machine

6.2 Conductor Losses

If total resistance of armature and field conductors is given, conductor loss can simply be calculated using I^2R, if current is known. Many times, conductor type and total weight of the conductor is given instead of total resistance of the conductor. In that situation, first, the conductor loss in *watt per kilogram* is calculated using (6.1), which is multiplied by the total conductor weight to get total conductor power loss in watt.

$$P_c = \frac{1000 J^2 \rho}{D} \tag{6.1}$$

In (6.1), J is the current density in *Ampere/square millimeter (A/mm^2)*, ρ is the resistivity of the conductor in *nano-Ohm meter (nΩ.m)*, and D is the density of conductor in *kilogram/cubic meter (kg/m^3)*.

The resistivity of a specific material can be calculated using (6.2)

$$\rho = \rho_o (1 + \alpha T) \tag{6.2}$$

Where ρ_o is the resistivity of the material at 0°C, α is the temperature coefficient of the resistance at 0°C, and T is the operating temperature in Celsius.

If length and area of cross section of the conductor is known, resistance of the conductor can be calculated (if required) using (6.3).

$$R = \frac{\rho L}{A} \tag{6.3}$$

The quantities ρ_o, α and D as required in (6.1) and (6.2) can be found in a standard table, as shown in *Table AX2* (*page 918 in the textbook)*. The resistivity of the most commonly used conductor, copper, is 15.88nΩ.m at 0°C, temperature coefficient at 0°C is 4.27×10^{-3}/0°C, and density is 8890kg/m^3.

6.3 Losses as a Function of Load

If there is no load connected to the machine, the armature current is either zero or quite small, based on the type of the machine. Hence, armature conductor loss is either zero or very small. In this situation, most of the loss is comprised of frictional, windage, and iron loss. If a machine has shunt field then there is also a shunt conductor loss.

TABLE AX2 ELECTRICAL, MECHANICAL AND THERMAL PROPERTIES OF SOME COMMON CONDUCTORS (AND INSULATORS)

Material	Chemical symbol or composition	Electrical properties: resistivity ρ 0°C nΩ·m	Electrical properties: resistivity ρ 20°C nΩ·m	Electrical properties: temp coeff at 0°C (× 10^{-3})	Mechanical properties: density kg/m³ or g/dm³	Mechanical properties: yield strength MPa	Mechanical properties: ultimate strength MPa	Thermal properties: specific heat J/kg·°C	Thermal properties: thermal conductivity W/m·°C	Thermal properties: melting point °C
aluminum	Al	26.0	28.3	4.39	2703	21	62	960	218	660
brass	≈ 70% Cu, Zn	60.2	62.0	1.55	≈ 8300	124	370	370	143	960
carbon/ graphite	C	8000 to 30 000	–	≈ −0.3	≈ 2500	—	—	710	5.0	3600
constantan	54% Cu, 45% Ni, 1% Mn	500	500	−0.03	8900	—	—	410	22.6	1190
copper	Cu	15.88	17.24	4.27	8890	35	220	380	394	1083
gold	Au	22.7	24.4	3.65	19 300	—	69	130	296	1063
iron	Fe	88.1	101	7.34	7900	131	290	420	79.4	1535
lead	Pb	203	220	4.19	11 300	—	15	130	35	327
manganin	84% Cu, 4% Ni, 12% Mn	482	482	±0.015	8410	—	—	—	20	1020
mercury	Hg	951	968	0.91	13 600	—	—	140	8.4	−39
molybdenum	Mo	49.6	52.9	3.3	10 200	—	690	246	138	2620
monel	30% Cu, 69% Ni, 1% Fe	418	434	1.97	8800	530	690	530	25	1360
nichrome	80% Ni, 20% Cr	1080	1082	0.11	8400	—	690	430	11.2	1400
nickel	Ni	78.4	85.4	4.47	8900	200	500	460	90	1455
platinum	Pt	9.7	10.4	3.4	21 400	—	—	131	71	1773
silver	Ag	15.0	16.2	4.11	10 500	—	—	230	408	960
tungsten	W	49.6	55.1	5.5	19 300	—	3376	140	20	3410
zinc	Zn	55.3	59.7	4.0	7100	—	70	380	110	420
air	78% N_2, 21% O_2	—	—	—	1.29	—	—	994	0.024	—
hydrogen	H_2	—	—	—	0.09	—	—	14 200	0.17	—
pure water	H_2O	—	2.5 × 10^{14}	—	1000	—	—	4180	0.58	0.0

As a machine is loaded and armature current increases, armature conductor loss also increases. The rest of the losses stay approximately same. The armature loss in a loaded armature contributes to the excess of heat, which may put a limit to the power supplied by the machine. Machine

temperature must not exceed the maximum allowable temperature of the insulation in the machine else windings will short out and machine will not operate.

Example 6.1

A DC machine is rotating at 900rpm and has armature winding that weighs 50Kg. Given current density is 3MA/m^2. Operating temperature is 90°C. Total iron loss is 1KW. Calculate (i) Copper loss, and (ii) Mechanical drag due to iron loss

(i) From *Table AX2*: Density of Cu = 88.90 Kg/m^2; ρ_o = 15.88nΩ.m; α = 4.27E-3 @ 0°C

Resistivity at 90°C: $\rho = \rho_o(1 + \alpha T)$ = 15.88(1 + 4.27E-3*90) = 21.988nΩ.m

Current density in A/mm^2: 3E6A/1E6mm^2 = 3A/ mm^2

Copper loss/Kg: $Pc = 1000J^2 \rho/D$ = (1000*3*21.98)/8890 = 22.252W/Kg

Total Copper loss: *Pc**50Kg = 1.11KW

(ii) Mechanical drag due to the iron loss is calculated in terms of torque due to the iron loss, as discussed earlier.

$P = nT/9.55 \rightarrow T = 9.55P/n$ = 9.55*1000/900 = 10.611N.m

6.4 Machine Efficiency

The efficiency of a machine is measured as the ratio of its output power to the input power. Generally, it is expressed in percentages.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{in} - P_{loss}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} \tag{6.4}$$

It is important to realize that efficiency of a motor in the absence of iron. frictional, and windage losses is always higher than if these losses are taken into consideration, since the motor has to supply extra current, and hence, extra power, to compensate for these losses. Although iron, frictional, and windage losses are part of the field and/or armature, for the sake of simplicity, these losses can be lumped together and shown in parallel to the supply. This will emphasize the fact that extra current has to be supplied by the source to compensate for these losses. A compound DC motor with iron, frictional and windage losses can be represented by the circuit model shown in *figure 6.2*.

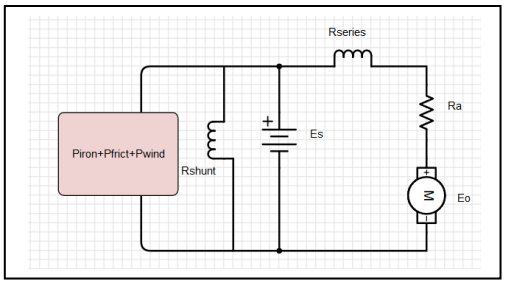


Figure 6.2: DC Compound motor with iron & frictional losses

A DC shunt motor with iron and frictional losses can be represented similarly except series field winding. Note that this model is only valid if the sum of the field and the armature current is not equal to the supply current. If the supply current is equal to the sum of field and armature currents, then do not show the iron and frictional losses in parallel, rather, they will be part of the armature and field, and they can be represented in series with the armature and/or field as resistive components to show the loss of power.

<u>*Example 6.2*</u>

A shunt motor with field resistance of 50Ω and armature resistance of 2Ω is operating at no load. Speed of rotation is 1000rpm. The motor is lap-wound with 100 slots and 6 turns/coil. Flux per pole is 0.1Wb. Armature current is 50A and field current is 10A. Friction and windage losses are 1100W and iron losses are 58W. Calculate efficiency of the motor.

Copper loss in the field: $P_{shunt} = I_f^2 R_f = 10^2*50 = 5KW$

Copper loss in the armature: $P_a = I_a^2 R_a = 50^2*2 = 5KW$

Induced voltage: $Eo = Zn\phi/60i = (100*6*2)(1000)(0.1)/60 = 2000V$

Output power: $Po = EoI_a = 2000*50 = 100KW$

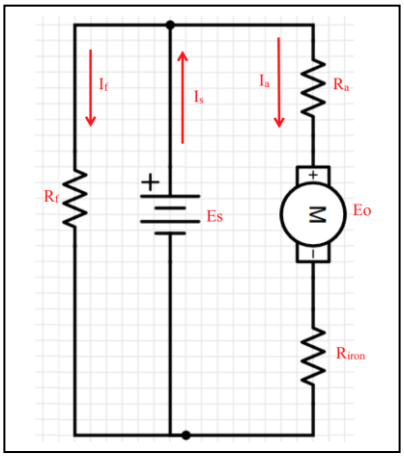
Total losses = 5KW + 5KW + 1.1KW + 58W = 11158W = 11.158KW

Efficiency: $\eta = Po/(Po + Losses) = 100/(100 + 11.158) = 0.899 = 89.9\%$

Example 6.3

A shunt motor has field resistance of 50Ω and armature resistance of 1Ω. It is connected to a 200V DC source. Armature current is 10A. Friction, windage, and iron losses are 50W and assume them to be part of the armature. Determine the efficiency of the motor.

Note: R_{iron} represents the resistor corresponding to the loss due to friction, windage, and iron.



Voltage drop across R_{iron}: $E_{iron} = P_{iron}/Ia =$ 5V

Induced Voltage: $E_o = E_s - I_a*R_a - E_{iron} =$ 185V

Source current: $I_s = I_a+I_f = 10 + 200/50 =$ 14A

Input Power: $Pi = Es*Is =$ 2800W

Output Power: $Po = Eo*Ia =$ 1850W

Efficiency: $\eta = Po/Pi*100 =$ 66.071%

6.5 Life Expectancy of Electric Equipment

The life expectancy of electrical equipment is limited by the temperature of its insulation. The higher the temperature, the shorter the life. Tests have shown that the service life of a machine is reduced by one-half every time the operating temperature rises by 10°C beyond the temperature threshold of its insulation. Hence, a class *A* machine that has a temperature threshold of 105°C and normal life expectancy of eight years will have service life of four years if it is operated at 115°C. If it is operated at 125°C, the service life will reduce to two years, and so on. Extremely low temperatures are also quite harmful as the insulation can freeze and crack. There are some special synthetic insulation materials that can operate at temperatures as high as 200°C and as low as -60°C.

6.6 Thermal Classification of Insulators

Thermal classification of different insulators is shown in the following table (*Ref: Wikipedia)*

Table 6.1: Classes of Insulation Systems

IEC 60085 Thermal class[3]	Old IEC 60085 Thermal class[3]	NEMA Class [4]	NEMA/UL Letter class	Maximum hot spot temperature allowed	Relative thermal endurance index (°C)[3]	Typical materials
90	Y			90°C	>90 - 105	Unimpregnated paper, silk, cotton, vulcanized natural rubber, thermoplastics that soften above 90 C [5]
105	A	105	A	105°C	>105 - 120	Organic materials such as cotton, silk, paper, some synthetic fibers [6]
120	E			120°C	>120 - 130	Polyurethane, epoxy resins, polyethylene terephthalate, and other materials that have shown usable lifetime at this temperature
130	B	130	B	130°C	>130 - 155	Inorganic materials such as mica, glass fibers, asbestos, with high-temperature binders, or others with usable lifetime at this temperature
155	F	155	F	155°C	>155 - 180	Class 130 materials with binders stable at the higher temperature, or other materials with usable lifetime at this temperature
180	H	180	H	180°C	>180 - 200	Silicone elastomers, and Class 130 inorganic materials with high-temperature binders, or other materials with usable lifetime at this temperature
200			N	200°C	>200 - 220	As for Class B,and including teflon
220		220	R	220°C	>220 - 250	As for IEC class 200
			S	240°C		Polyimide enamel (Pyre-ML) or Polyimide films (Kapton and Alconex GOLD)
250				250°C	>250	As for IEC class 200. Further IEC classes designated numerically at 25 °C increments.

6.7 Speed and Size of a Machine

Although the nominal power rating of a machine depends upon the maximum allowable temperature rise but its size depends upon the power and speed of rotation. This, in turn, results in the size of motor depending upon torque that it develops. Thus, a 100KW motor running at 2000 rpm is approximately the same size as a 10KW motor running at 200 rpm as they develop the same torque. This results in low-speed motors to be quite costly as compared to high-speed motors to produce the same power. Hence, if a load requires low-speed, generally it is preferred to use a high-speed motor coupled with a gear to turn the load at low speeds rather than connecting load directly to a low-speed motor.

PROBLEMS

1. A DC shunt motor with field resistance of 110Ω and total armature resistance of 0.6Ω has 17A armature current. It is connected to 110V, 20A line. CEMF is 100V, calculate,
   (i) Total copper losses (I^2R losses)
   (ii) Total mechanical + iron losses
   (iii) Efficiency

2. A DC compound motor has shunt field resistance of 500Ω and series field resistance of 2Ω. Shunt field current is 0.44A. Armature is lap wound with 200 conductors running at 600rpm. Flux per pole is 0.1Wb. Armature winding resistance is 0.1Ω and it is dissipating 10W. Other losses amount to 35W. Calculate the efficiency of the motor.

3. A DC shunt motor is connected to a 20V source. Shunt resistance is 50Ω. CEMF is 15V and output power is 20W. Windage, friction, and iron losses are part of the armature and they are 4.533W. Calculate:

   (i) Total conductor losses
   (ii) Efficiency

4. A DC shunt motor is connected to a 100V source. It has 50Ω shunt resistor and 1.5Ω armature resistor. Armature current is 5A. Friction and iron losses are 40W. Calculate:

(i) Total conductor loss
(ii) Output power and efficiency

5. A DC compound motor has shunt field winding resistance of 50Ω, series field winding resistance of 2Ω, and armature resistance of 0.5Ω. Armature current is 15A and source voltage is 110V. Iron, windage, and frictional losses are 100W. Calculate the efficiency of the motor.

6. A compound motor has 240Ω shunt field and 4Ω series field with 10 turns. Series field mmf is 40A.t. Armature resistance is 0.5Ω. Frictional and iron losses are 50W and they can be assumed to be part of the armature. Motor is connected to a 240V DC source. Calculate:

(i) Total copper losses

(ii) Output power

(iii) Efficiency

Chapter 7 – Active, Reactive, and Apparent Power

7.1 Different Types of Power

In a power system, there are different types of power with different units. These power types include:

(i) Instantaneous Power
(ii) Active Power
(iii) Reactive Power
(iv) Complex Power
(v) Apparent Power

For any power system and any load, values for all power types can be calculated, although many times it is either not required or already known. Description of each power type is given in the following sections.

7.2 Instantaneous Power

When voltage and current are a function of time, their product gives *instantaneous power*, i.e. power at any instant of time. Let *v* be the voltage applied across some device and current *i* flows through it, as shown in *figure 7.1*,

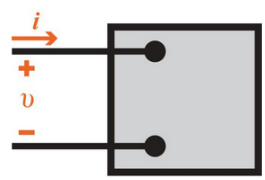


Figure 7.1: Voltage 'v' applied to a device requiring current 'i'

If both voltage and current are sinusoidal in nature, they can be represented by the following generic time-domain expressions:

$$\begin{aligned} v(t) &= V_p cos(\omega t + \theta) \quad \text{(Volt)} \\ i(t) &= I_p cos(\omega t + \phi) \quad \text{(Ampere)} \end{aligned} \tag{7.1}$$

where V_p and I_p are the peak values of voltage and current waveforms, and θ and ϕ are their corresponding phase angles, respectively.

The instantaneous power supplied to the device can be given by:

$$p(t) = v(t)i(t) = VpIp\cos(\omega t + \theta)\cos(\omega t + \phi) \text{ (Watt)} \tag{7.2}$$

The unit of instantaneous power is *Watt*.

Analysis of electrical networks with sinusoidal signals is generally done in complex frequency *phasor* domain instead of time domain as it converts integro-differential circuit equations into simple algebraic equations with complex numbers. For the voltage and current given in (7.1), the equivalent phasor quantities will be given by:

$$\begin{aligned} \mathbf{V}(j\omega) &= V_p\angle\theta \\ \mathbf{I}(j\omega) &= I_p\angle\phi \end{aligned} \tag{7.3}$$

Note that the signals are still time-domain signals but for the sake of analysis in the complex frequency domain, they are written as complex numbers in polar form where magnitude of the signal is the peak value and angle is the phase of the signal. Equivalently, the complex frequency notations can also be given in Cartesian form as complex numbers. Once analysis of the circuit is done in the complex frequency domain, the result is converted back in time domain by writing it in the form $Acos(\omega t+B)$, where A is the peak value and B is the phase of the resultant quantity.

The device as shown in *figure 7.1* is associated with an impedance which can be calculated by dividing the phasor quantity of the voltage by the phasor quantity of the current given in (7.3),

$$\mathbf{Z}(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} = \frac{V_p\angle\theta}{I_p\angle\phi} = \frac{V_p}{I_p}\angle\theta - \phi \tag{7.4}$$

where the magnitude of the impedance is Vp/Ip and its phase is $\theta-\phi$.

7.3 Active Power

Active power has many other names, real power, average power, DC power etc. This is the average power consumed in a device or supplied by a source over one cycle. To calculate average or active power, calculate the average of $p(t)$ given in (7.2) over one time period, as shown below,

$$
\begin{aligned}
P &= \frac{1}{T}\int_{t=0}^{T} p(t)dt = \frac{1}{T}\int_{t=0}^{T} VpIp\cos(\omega t+\theta)\cos(\omega t+\phi)dt \\
&= \frac{1}{2\pi}\int_{\omega t=0}^{2\pi} VpIp\cos(\omega t+\theta)\cos(\omega t+\phi)d\omega t \\
&= \frac{VpIp}{2\pi}\int_{\omega t=0}^{2\pi} \frac{\cos(2\omega t+\theta+\phi)+\cos(\theta-\phi)}{2}d\omega t \\
&= \frac{VpIp}{2}\cos(\theta-\phi) = \frac{Vp}{\sqrt{2}}\frac{Ip}{\sqrt{2}}\cos(\theta-\phi) = V_{rms}I_{rms}\cos(\theta-\phi)
\end{aligned}
\tag{7.5}
$$

Hence, active power is given by $V_{rms}I_{rms}\cos(\theta-\phi)$ or $V_{rms}I_{rms}\cos\angle\mathbf{Z}$. The unit of active power is *Watt*. This is the power that produces some tangible quantity. For example, conversion of electrical energy into light energy, mechanical energy, heat energy etc. Also, this is the energy that is measured by kilowatt-hour meters and consumers are charged for using it.

Active power is the power that is dissipated in only the resistive part of the network. It is not the power associated with inductors and capacitors, i.e., the reactive part of the electric network. Hence, if there are multiple resistors in a network, total active power supplied is the sum of power dissipated in each resistor,

$$
P = V_{rms}I_{rms}\cos\angle\mathbf{Z} = I_1{}^2R_1 + I_2{}^2R_2 + \cdots + I_n{}^2R_n = \frac{V_1^2}{R_1} + \frac{V_2^2}{R_2} + \frac{V_3^2}{R_3} + \cdots + \frac{V_n^2}{R_n} \tag{7.6}
$$

where I_n is the rms value of the current and V_n is the rms value of the voltage across *n*-th resistor. Be very careful if you are using V^2/R formula to calculate active power as the value of the rms voltage used is the voltage across the resistor only, not the whole load.

If a 10V (peak) sinusoidal source is applied across a 1Ω resistor, it will produce a sinusoidal current with a peak value of 1A. When the voltage and current waveforms, which are in phase, are multiplied by each other, instantaneous power waveform is generated. All three waveforms are shown in *figure 7.1*. Observe that the power (green) waveform is unidirectional, i.e., above the *x*-axis. Hence, an average value of this power exists, which is given by (7.5) or (7.6). This is the active power supplied by the source or dissipated in the load resistor.

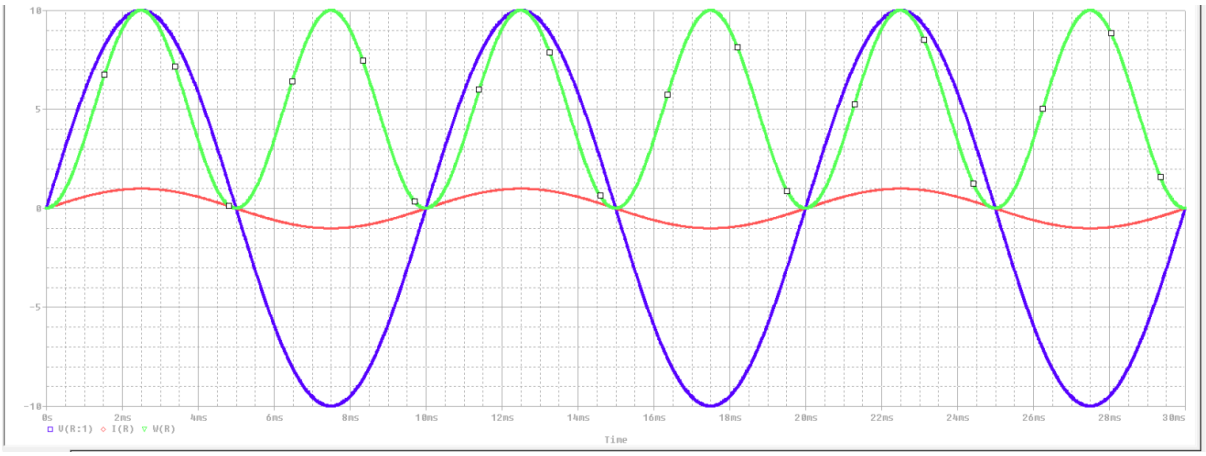


Figure 7.1: Voltage Waveform (blue), Current Waveform (red), and Power Waveform (green) in a 1Ω Resistor

7.4 Reactive Power

As it was discussed in the previous section, average or active power supplied by a source is consumed in the resistive part of the network. This is the power that is measured through kilowatt-hour meters, and everyone pays bill to consume it. In the study of generators and motors it was discussed that current is required to setup magnetic field in a machine through coils (inductors). The power that is used to set up this magnetic field is not the active power as it is not delivered to the resistive part of the circuit, rather it is the *reactive power*. In general, reactive power is the power related to the reactive part (inductor and capacitor) of the circuit. This power cannot be measured through kilowatt-hour meters but there are special meters, called VAR meters to measure it.

The reactive power supplied or consumed in a reactive component can be given by,

$$Q = V_{rms} I_{rms} \sin(\theta - \phi) \tag{7.7}$$

The unit of measuring reactive power is *Volt-Ampere Reactive (VAR)*. Observe that the current lags the voltage by 90° in an inductor and leads by 90° in a capacitor. Hence, reactive power absorbed by an inductor is positive, since $\theta - \phi$ is positive, and it is negative for a capacitor since $\theta - \phi$ is negative. Therefore, on average, an inductor absorbs reactive power, and a capacitor supplies reactive power. In other words, in phasor (complex frequency) domain, current goes in an inductor and comes out of a capacitor.

Magnitude of the reactive power in an inductor or capacitor can also be measured by applying Ohm's law on the basic power expression as follows:

$$|Q| = I_{rms}^{2} X = \frac{V_{rms}^{2}}{X} \tag{7.8}$$

where X is the magnitude of the reactance. Note that the reactive power calculated through (7.8) should be appended by a negative sign if capacitive reactance is used to show that the reactive power is coming out of the capacitor.

Voltage, current, and power waveforms through a capacitor and an inductor are shown in *figure 7.2(a)* and *(b)* respectively. Observe that the current is leading the voltage by 90° in a capacitor (current is maximum when voltage is zero) and lagging 90° in an inductor (current is minimum when voltage is zero). In any case, the power waveform is a pure sinusoid; hence, average power is zero. Therefore, as discussed earlier, there is no average or active power associated with reactive components.

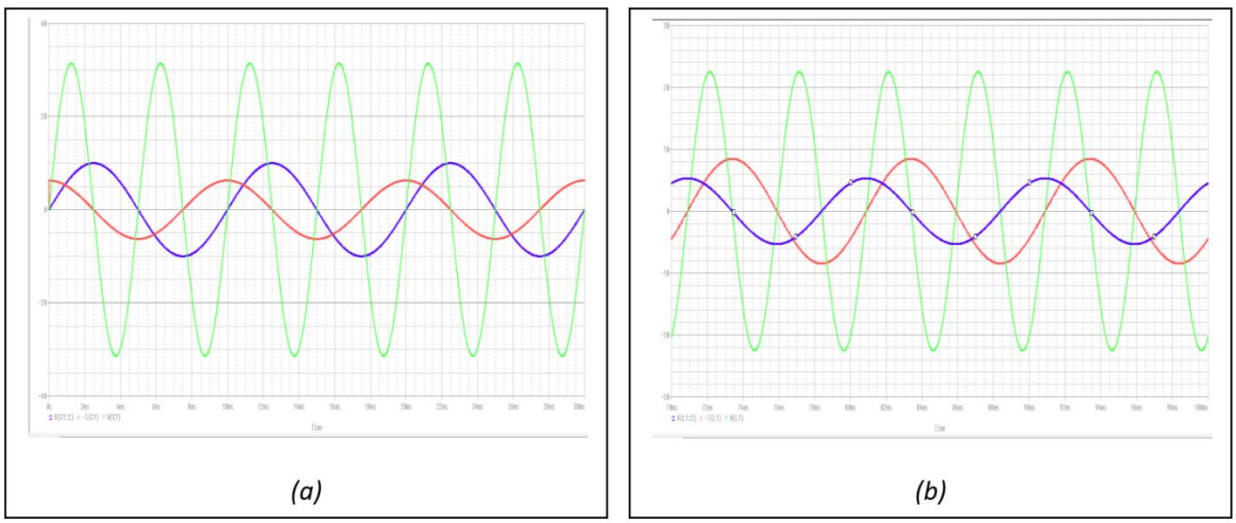


Figure 7.2: Voltage (Blue), Current (Red), and Power (Green) Waveforms across Capacitor (a) and Inductor (b)

7.5 <u>Current Components</u>

Let $v(t) = V_m cos(\omega t)$ be applied across an impedance and current $i(t) = I_m cos(\omega t + \phi)$ flows through it. In phasor domain, the time-domain quantities can be written as $V_m \angle 0$ and $I_m \angle \pm\phi$. Observe that the current in phasor domain has two components; $I_m cos(\phi)$ and $I_m sin(\phi)$, as shown in *figure 7.3*

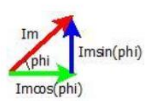


Figure 7.3: Components of current

The two current components, $I_m cos(\phi)$ and $I_m sin(\phi)$, as shown in *figure 7.3*, are called *in-phase* (I_p) and *quadrature* (I_q) components of current, respectively. Let's revisit the expressions for active and reactive powers:

$$P = \frac{V_m I_m}{2}\cos(-\phi) = \frac{V_m I_m}{2}\cos(\phi) = \frac{V_m I_p}{2}$$
$$Q = \frac{V_m I_m}{2}\sin(-\phi) = -\frac{V_m I_m}{2}\sin(\phi) = -\frac{V_m I_q}{2} \tag{7.9}$$

Hence, the active power given by the source is calculated by the product of voltage and the in-phase current component and reactive power is the product of voltage and the quadrature component.

7.6 Complex Power

Complex power is the product of the rms components of phasor voltage and complex conjugate of phasor current. It is calculated as follows:

$$\begin{aligned}\mathbf{S} &= \mathbf{V}_{rms}\mathbf{I}_{rms}^{\ *} = V_{rms}\angle\theta(I_{rms}\angle\phi)^* = (V_{rms}\angle\theta)(I_{rms}\angle-\phi) = V_{rms}I_{rms}\angle(\theta-\phi) = S\angle(\theta-\phi)\\ &= V_{rms}I_{rms}\cos(\theta-\phi) + jV_{rms}I_{rms}\sin(\theta-\phi)\\ &= P \pm jQ\end{aligned} \tag{7.10}$$

Observe that phase $\theta-\phi$ is the phase of the impedance in which power is consumed. Complex power can be represented into its active and reactive components by a *power triangle*, as shown in *figure 7.4*.

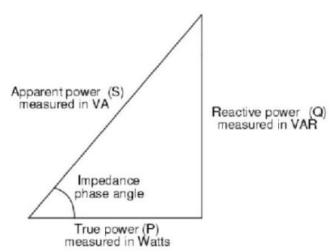


Figure 7.4: Power Triangle

The unit of complex power is *Volt-Ampere (VA)*, and this is the only power for which your calculated result must be a complex number.

7.7 Apparent Power

Apparent power is the magnitude of the complex power. It is measured in *Volt-Ampere (VA)*. From (7.10), apparent power is given by:

$$S = V_{rms} I_{rms} \tag{7.11}$$

Observe that the apparent power appears in the expressions of active and reactive power, as given by (7.5) and (7.7); hence, another way to calculate it is as follows:

$$S = \frac{P}{\cos(\theta - \phi)} = \frac{Q}{\sin(\theta - \phi)} \tag{7.12}$$

Example 7.1

A Wattmeter and a VARmeter are connected to an AC line with 155.56V peak value. An AC motor is connected to the line for which the wattmeter is recording 2000W and the VARmeter is recording 1000VAR. Calculate:

(i) In-phase and Quadrature components of the motor current:

>>Es_{peak} = 155.56; $Es_{RMS} = Es_{peak}/\sqrt{2}$ = 110V

In-phase component: $Ip = P/Es_{RMS}$ = 18.182A

Quadrature component: $Iq = Q/Es_{RMS}$ = 9.0911A

(ii) Total current: $I = \sqrt{Ip^2 + Iq^2}$ = 20.328A

(iii) Apparent Power: $|S| = E_{RMS} * I$ = 2236.1VA

(iv) Phase angle between voltage and current or impedance angle of the motor:

$\angle V - \angle I = \angle Z = \tan^{-1}(Q/P)$ = 26.565°

7.8 Power Factor

Power factor is an extremely important quantity in power systems. It is the quantity that describes the type of load and the amount of bill that we pay to the electrical company. Power factor is the cosine of the phase of load impedance,

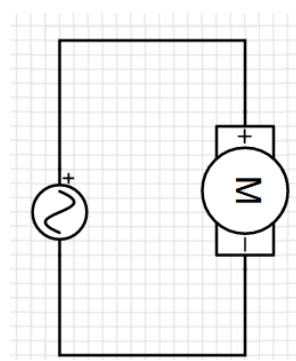
$$PF = \cos(\theta - \phi) \tag{7.13}$$

Hence, the power factor is unity if the load is pure resistive (voltage and current are in phase) and zero if the load is pure reactive (voltage and current are perpendicular to each other). For real loads, power factor is between zero and one. Remember that the active power is measured by $S*\cos(\theta-\phi)$ or $(S)(PF)$; hence, active power is maximum when the power factor is unity or the load is pure resistive, and it is zero if the load is pure reactive because the power factor is zero. Therefore, when the power factor is unity, the power company gets payment for the full amount of power supplied to the consumer and if the power factor is zero, consumer pays nothing despite receiving reactive power (i.e., load current) from the power company. For real loads, power companies adjust the bill based on the *power factor correction* to receive the fair amount of compensation for the energy supplied by them.

Power factor is generally represented in percentage (0-100%). Also, based on the load type (capacitive or inductive), it is represented as *leading* or *lagging* power factor. If current leads voltage for the load, i.e., if the load is capacitive, power factor is leading else it is lagging (for inductive loads).

Example 7.2

A motor draws 10A from 110V (rms), 60Hz line with the power factor of 80% lagging. Determine the active power absorbed by the motor and the reactive power supplied by the line.



(i) Active power absorbed by the motor:

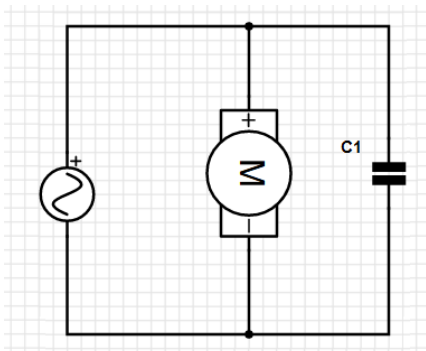
$P = Es*I*PF =$ 880 W

(i) Reactive power supplied by the line which will be absorbed by the motor:

$Q = Es*I*\sin(\cos^{-1}(PF)) =$ 660VAR

Example 7.3

A 50uF capacitor is added across the motor from Example 7.2 to reduce the burden of current on the line by supplying a portion of reactive power to the motor. Determine the new reactive power supplied by the line, line current, apparent power, and power factor of the line.



Since motor is the same as in the *Example 7.2*, it will still Require 880W and 660VAR of power at 80% PF lagging. The reactive power associated with the capacitor is negative, i.e. on average, it supplies reactive power.

First, we will calculate the magnitude of the reactance of the capacitor: $Xc = 1/(2*\pi*f*C) = 53.052\Omega$.

Reactive power of the capacitor: $Qc = -Es^2/Xc = -228.08$VAR

>>Therefore, total reactive power supplied by the line: $Qs = Q_M + Qc =$ 431.92VAR

>> Apparent power of the line: $|S| = \sqrt{Ps^2 + Qs^2} =$ 980.28VA

Note that the active power supplied by the line is consumed in the motor, hence, $Ps = P_M$

>> Line current: $I = |S|/Es =$ 8.9117A.

Note that the line current is reduced from 10A to 8.91A.

>> Line power factor: $PF = P/|S| = 0.897$ lagging or 89.7% lagging (improved).

7.9 Active Sources and Loads

If voltage and current are in phase and current is entering in the positive terminal of a device, it is an active load else it is an active source. Likewise, if voltage and current are perpendicular to each other and current is entering in the positive terminal of a device, it is a reactive load else it is a reactive source. In general, anytime positive current comes out of the positive side of the power source, it is an active (or reactive) source else it is an active (or reactive) load.

7.10 System Comprising of Several Loads

A power system is comprised of several loads, some active and some reactive. The total active power supplied by the power system is the sum of the active power consumed in all of the active loads. Likewise, the total reactive power supplied by the system is the net reactive power consumed by all of the reactive components. Keep in mind that the reactive power to inductive loads is considered positive, i.e., positive current flows in the positive terminal of the inductive component, whereas reactive power to the capacitive loads is negative, i.e., positive current comes out of the positive terminal of the capacitive components. Hence, net reactive power supplied by the system is the difference between the power supplied to the inductive part and received from the capacitive part.

Example 7.4

A power system with Es = 220V(rms) is furnishing the following loads:

Resistive: 8KW, 14KW, and 2KW

Inductive: 7KVAR, 8KVAR, and 5KVAR

Capacitive: 16KVAR and 9KVAR

Calculate:

(i) Total active power absorbed by the load: P = 8+14+2 = 24KW

(ii) Total reactive power supplied by the line: $Q = Q_L + Qc$ = (7 + 8 + 5) – (16 + 9) = -5KVAR

Therefore, line is receiving 5KVAR of reactive power from the load.

(iii) Apparent power of the system: $|S| = \sqrt{P^2 + Q^2}$ = 24.515KVA

(iv)Line current: $I = |S|/Es|$ = 111.43A

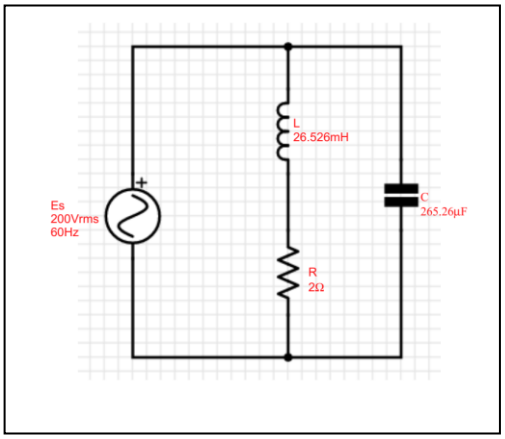
(v) Power factor of the system: $PF = P/|S| = 0.978$ leading or 97.8% leading.

Note that since the total reactive power is negative, source sees load as a capacitive load for which power factor is leading, since current leads voltage in capacitive loads.

Example 7.5

A motor is shown with its resistance in series with the inductance. A capacitor is placed across the motor to improve its power factor. Calculate:

(i) Complex power of the motor:



First, we will determine the reactance of the motor

Inductance: $X_M = j\omega L = 10i\Omega$

Motor impedance: $Z_M = R + X_M = 2+10i\Omega$

Motor current: $I_M = Es/Z_M = 3.8462 - 19.2308i$ A

Complex power of the motor: $S = Es*I_M^*$

S = 769.23 + 3846.15i VA

Hence, the active power of the motor is 769.23W and the reactive power is 3.846KVAR.

(ii) Reactive power of the capacitor:

First, we will determine the reactance of the capacitor: $Xc = 1/(j\omega C) = -10i\Omega$

Capacitor current: Ic = Es/Xc = 20i A

Reactive power of the capacitor: $Qc = Es*Ic*\sin(\angle Es - \angle Ic) = 200*20*\sin(0 - 90^o) = -4KVAR$

(iii) Total reactive power of the line: $Q = Q_M + Qc = 3846.15 - 4000 = -153.85VAR$

(iv) Apparent power of the line: $|S| = \sqrt{P^2 + Q^2} = 784.46VA$

(v) Line current: $I = |S|/Es = 3.9223A$

(vi) Power factor of the line: $PF = P/|S| = 0.980$ leading

PROBLEMS

Note: RMS values of all voltages and currents are given. If in any problem, peak values are given then convert them into RMS first before you calculate active, reactive, apparent, and complex powers.

1. A motor draws 20A from 110V, 60Hz line with power factor of 77% lagging, calculate,
   (i) Active power absorbed by the motor
   (ii) Reactive power supplied by the line
   (iii) In-phase current component
   (iv) Quadrature current component
   (v) Resistance of the motor
   (vi) Reactance of the motor
   (vii) Impedance of the motor

2. If a 1000 uF capacitor is placed at the terminals of the motor in problem # 1 to reduce the amount of current, what will be the new value of line current and new PF of the motor? Assume that motor still requires same active and reactive power as being calculated in the last problem.

3.

For the circuit of Fig. 11.39, compute the average power delivered to each load, the apparent power supplied by the source, and the power factor of the combined loads if (*a*) $\mathbf{Z}_1 = 14\angle 32^\circ\ \Omega$ and $\mathbf{Z}_2 = 22\ \Omega$; (*b*) $\mathbf{Z}_1 = 2\angle 0^\circ\ \Omega$ and $\mathbf{Z}_2 = 6 - j\ \Omega$; (*c*) $\mathbf{Z}_1 = 100\angle 70^\circ\ \Omega$ and $\mathbf{Z}_2 = 75\angle 90^\circ\ \Omega$.

Note: average power is same as active power

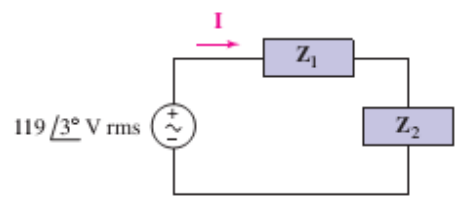


FIGURE 11.39

4. Two loads are connected to your standard AC power outlet (110V, 60Hz). The first load has a 60% leading power factor and pulling 30KVA from the outlet. The second load is pure resistive and pulling 20KW from the outlet. Determine:
   (i) Total active and reactive power drawn from the outlet.
   (ii) Values of the induvial components of each load.

5. Two AC motors are connected to 110V (rms), 60Hz standard power outlets. Motor # 1 can be represented by a 5Ω resistor in series with a 30mH inductor. There is also a 250μF capacitor connected in parallel to the motor to correct its power factor. Motor # 2 can be represented by a 10Ω resistor connected in parallel with a 10Ω inductive reactance. Determine **total current**, **total active and reactive power**, and **power factor** of the line.

6. Two loads are connected to your standard 110V(rms), 60Hz outlet.

   Load 1: R = 200Ω in series with L = 0.5H

   Load 2: R = 100Ω in parallel with C = 10μF

   Calculate:

   (i) Active and reactive power of load 1
   (ii) Power factor of load 1
   (iii) Active and reactive power of load 2
   (iv) Power factor of load 2
   (v) Current supplied from the source and power factor of the source

Chapter 8 – Three-Phase Circuits

Electrical Power is generated, transmitted, and distributed in the form of three-phase power. There are certain advantages of a three-phase power system over a single-phase system:

- Three-phase systems are not much complicated as compared to a single-phase system. They are cheaper and more efficient.
- Three-phase transmission lines can deliver more power for a given weight and cost.
- Voltage regulation for three-phase transmission lines is better.

In U.S.A, homes usually have a single-phase system; it is one of the phases of the three-phase system stepped down to 110V(rms)/120V(rms) with a center-tapped step-down transformer. Homes in many other countries get all three phases coming to them with different loads distributed on different phases.

Note that some of the appliances in our homes require 220V(rms)/240V(rms) instead of 110V(rms)/120V(rms). This is done by connecting the two 180° out-of-phase terminals of the center-tapped transformer to create a 220V(rms)/240V(rms) voltage socket, as shown in the following figure. Observe that this circuit doesn't have a neutral wire, rather two same 180° out-of-phase hot (phase) wires between which the load will be connected. It is still a single-phase system.

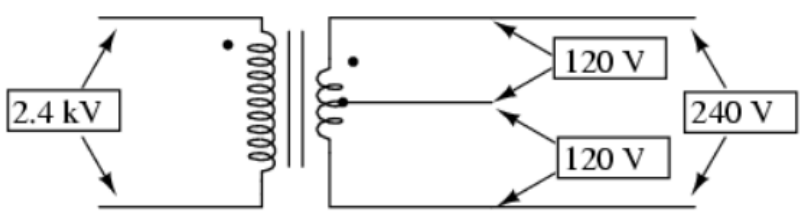


Figure 8.1: Center-tapped step-down transformer that provides power to homes

A three-phase system provides voltage between three lines that have the same peak and root-mean-square (rms) values but they are 120° out-of-phase with each other. Hence, the instantaneous value of the voltage for each line is different. A common depiction of a three-phase system and its voltage waveforms is shown in *figure 8.2*. In the figure, V_A, V_B and V_C are three line-to-neutral voltages, also called *phase* voltages, and V_{AB}, V_{BC}, and V_{CA} are line-to-line voltages, also referred to as *line* voltages.

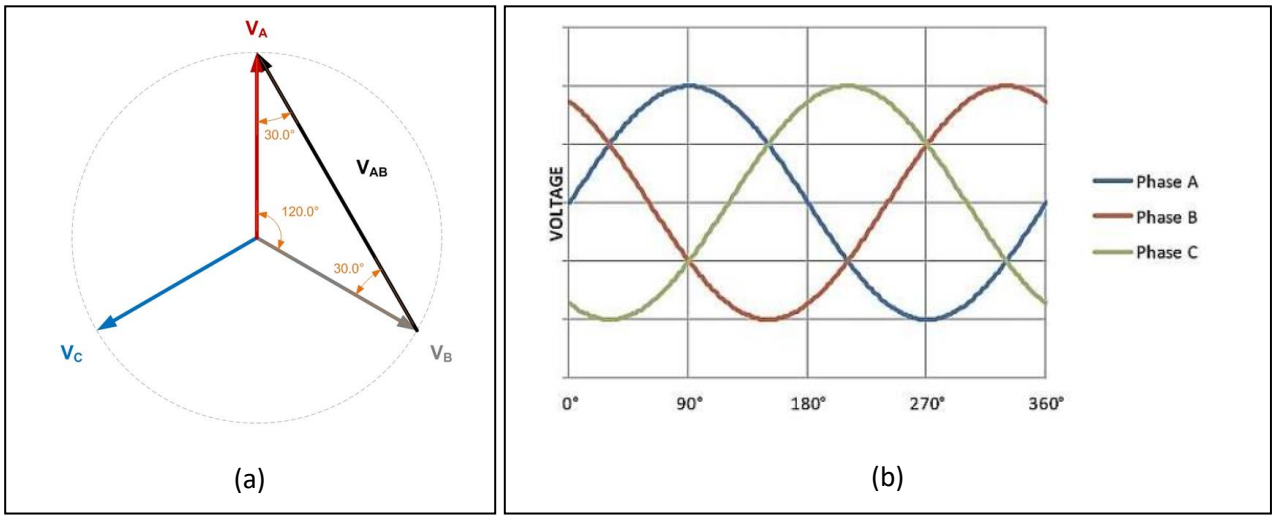


Figure 8.2: (a) Representation of a Three-phase System (b)Phase Voltages

8.1 Single-Phase Generator

To appreciate the constructional advantage and characteristics of a three-phase system, it is important to compare it against single-phase and two-phase generator systems. The basic AC generator that we discussed in chapters 2 and 4 is a single-phase generator. However, commercial AC generators generally have rotating magnetic field and static conductors in which voltage is induced. This arrangement of rotating magnet (rotor) and static voltage inducing conductors (stator) yield *alternating current generators* or *synchronous generators*. Detailed characteristics of synchronous generators will be studied in a later chapter. In this chapter, we will use the generator concepts that we have developed so far, simply to determine the output of synchronous generators.

Observe the arrangement of a single-phase synchronous AC generator with a permanent magnet as the rotor and conductor coils on stator, as shown in *figure 8.3*,

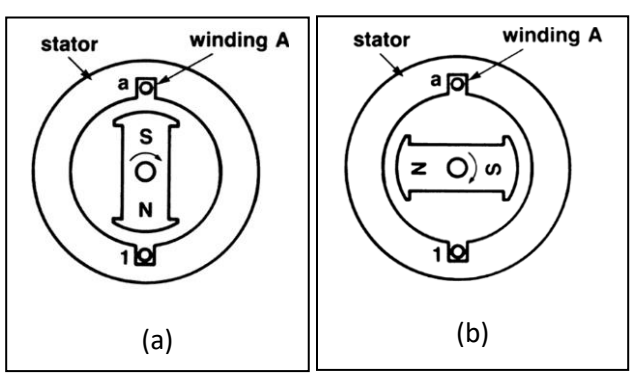


Figure 8.3: Construction of a simple single-phase AC generator

Note that we can use the equation **l.(v** x **B)** to calculate the value of the induced voltage in the coil. However, care should be taken about the direction of the velocity. In the expression **l.(v** x **B), v** is the velocity of the armature or conductor coils. In the AC generator under discussion, conductors are stationary, but magnet is moving. Hence, the velocity of conductors becomes *relative velocity* with respect to the magnet. Assume that you are sitting on the magnet which is rotating with a velocity of **v** in the counterclockwise direction. When you look at the conductors from the magnet, it will appear to move in the opposite, i.e., clockwise direction. Hence, relative velocity of conductors will always be opposite to the direction of the rotation of magnet and that direction will be used in the expression to calculate the induced voltage and its direction.

In *figure 8.3(a)*, the relative velocity of side *a* of the conductor is –**j**, the direction of the magnetic flux is –**k**, hence **v** x **B** will yield a direction of **i.** Therefore, polarity of the induced voltage in side *a* of the conductor will be positive with respect to side *1*, and if shorted, current is going to come out of side *a* and will go into side *1*. When the magnet rotates to 90°, as shown in *figure 8.3(b)*, flux will not cut the conductors and hence, the induced voltage will be zero. The output waveform for one cycle is usually *near* sinusoid, but for the sake of simplicity, we will assume the output to be a pure sinusoidal waveform, as shown in *figure 8.4*.

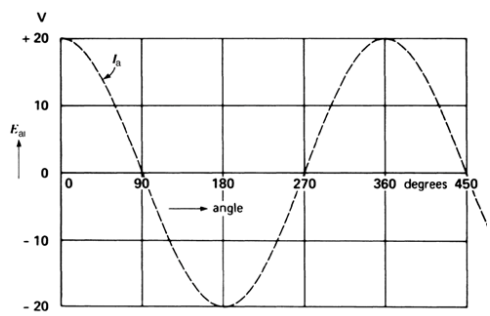


Figure 8.4: Output of a single-phase AC generator

If a resistor is connected to the terminals of the generator, the average or real power supplied will be,

$$P = V_{rms}I_{rms} = \frac{V_p I_p}{2} \text{ Watt} \tag{8.1}$$

8.2 Two-Phase Generator

The only difference between a single-phase and a two-phase generator is that there are two separate sets of windings on the stator, as shown in *figure 8.5*. These windings are isolated from each other, and two separate sets of voltages are induced in them, which have a phase difference of 90° between them.

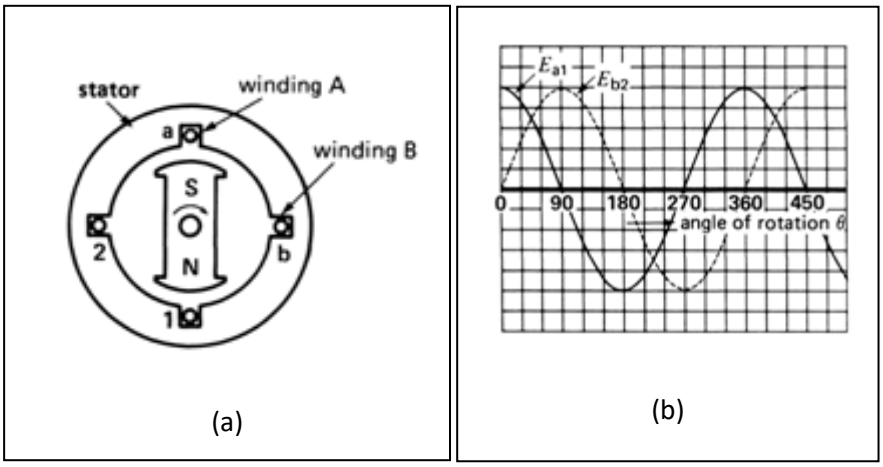


Figure 8.5: (a) Two-phase AC generator (b) Output voltage waveforms for the two phases

The voltage expressions for the two phases can be given by:

$$E_{a1} = Vp\cos(\omega t)\,\text{V}$$
$$E_{b2} = Vp\cos(\omega t - 90^{\circ}) = Vp\sin(\omega t)\,\text{V} \tag{8.2}$$

When a load resistor is connected to each of the phases (assume same resistor value), each phase will give a power output of $V_{rms}I_{rms}$. Hence, total output power of the two-phase generator will be,

$$P = 2V_{rms}I_{rms} = V_pI_p \text{ Watt} \tag{8.3}$$

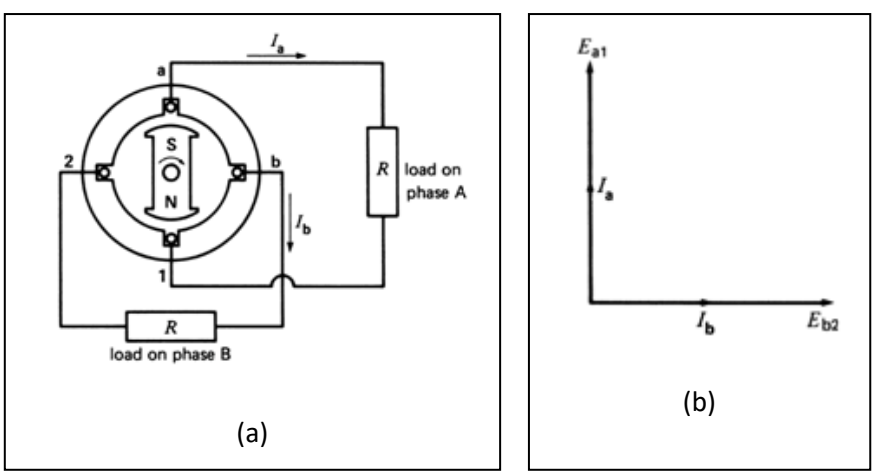


Figure 8.6: (a) Loads connected to a two-phase generator (b) Phasor diagram of voltages and currents

Thus, a two-phase AC generator will be able to supply double the amount of power as compared to a single-phase generator at a relatively small cost of adding an extra set of stator winding. The mechanical power required to rotate the magnet (rotor) as well as the size of the generator is the same as a single-phase system. A two-phase generator has less vibration as compared to a single-phase, therefore, they are less noisy.

Example 8.1

A two-phase generator rotates at 6000 rpm and the RMS voltage generated is 110V per phase. Determine: (i) Peak voltage per phase (ii) Output frequency in Hz (iii) Time interval between the peaks of two phases

(i) Peak Voltage: $V_{peak} = \sqrt{2}*110 = 155.56V$

(ii) Output frequency: f = *rotation per second* = $n/60$ = 100Hz

(iii) Time period: $T = 1/f = 10ms$

Phase difference between the two peaks is 90°, which is one-fourth of the full cycle.

Therefore, time interval between the two peaks will be one-fourth of the time period = 1/4*T = 2.5ms.

8.3 Three-Phase Generator

If another set of winding is added to the stator of a two-phase generator, which is separated from the other two sets, it results in a three-phase generator. Once again, the only added cost is an extra set of winding; mechanical power required to rotate the magnet as well as size of the generator remains the same. The induced voltage in the three sets of windings is 120° shifted from one another. *Figure 8.7* shows the construction of the generator as well as the three induced voltages.

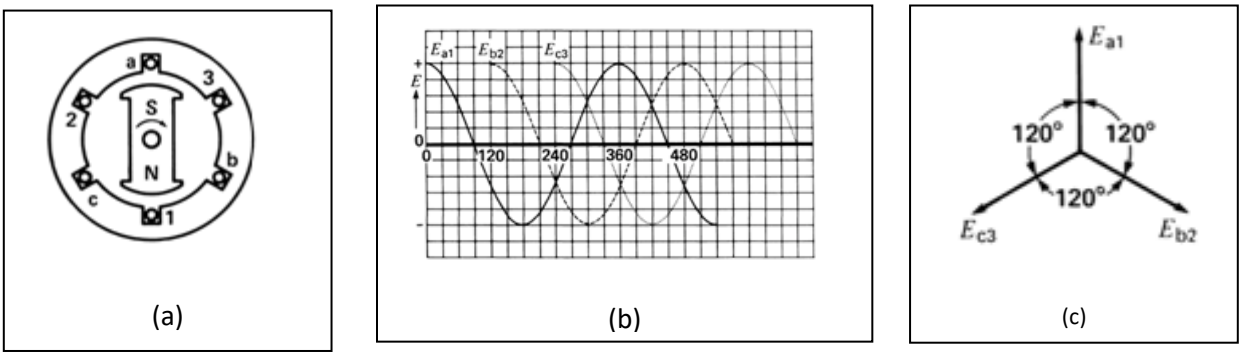


Figure 8.7: (a) 3-Phase AC generator (b) Voltage waveforms for the three phases (c) Phasor relationship between the three phases

The three phase voltages can be given by the following expressions:

$$E_{a1} = Vp\cos(\omega t)\,\text{V}$$
$$E_{b2} = Vp\cos(\omega t - 120^\circ) \qquad (8.4)$$
$$E_{c3} = Vp\cos(\omega t - 240^\circ) = Vp\cos(\omega t + 120^\circ)$$

Total average power for the three-phase generator is the sum of the average power of each phase, which is $V_{rms}I_{rms}$. Therefore, the total average power is given by,

$$P = 3V_{rms}I_{rms} = \frac{3}{2}V_pI_p \text{ Watt} \qquad (8.5)$$

Hence, the total average power output is 1.5 times the output of a single-phase generator, which is a great gain at the expense of a little cost of additional set of conductors on the stator.

8.4 Circuit Representation of a Three-Phase System

There are three circuit representations of a three-phase generator; *six-wire* circuit, *four-wire* circuit (most commonly used), and *three-wire* circuit.

Six-wire System: This is the original representation of a three-phase generator shown in *figure 8.7(a).* The equivalent circuit is shown in *figure 8.8.*

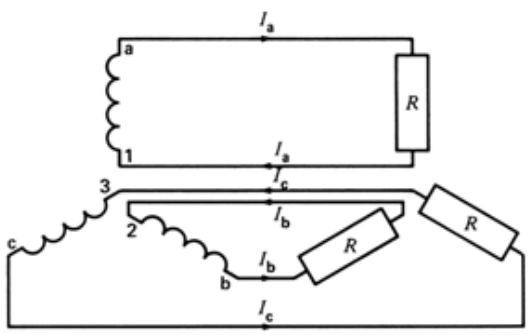


Figure 8.8: Six-Wire Three-Phase System

To reduce the number of conductors, a single wire is used as return, which is called *neutral*. This results in a four-wire system, as shown in *figure 8.9.*

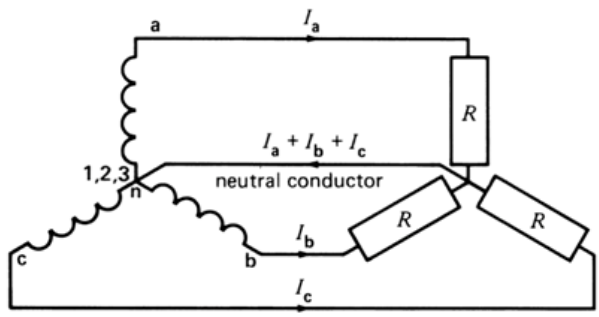


Figure 8.9: Four-Wire three-Phase System

For a balanced system, i.e., equal load on each phase, as shown in *figure 8.9*, sum of the three phase currents will be equal to zero, as can be shown by (8.6)

$$\begin{aligned}\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c &= \frac{\mathbf{V}_{an}}{R} + \frac{\mathbf{V}_{bn}}{R} + \frac{\mathbf{V}_{cn}}{R} \\ &= \frac{V_p}{R} + \frac{V_p\angle -120^o}{R} + \frac{V_p\angle -240^o}{R} \\ &= I_p + I_p\angle -120^o + I_p\angle -240^o \\ &= I_p(1 - 0.5 - j0.866 - 0.5 + j0.866) = 0\end{aligned} \tag{8.6}$$

Since there is no current in the neutral wire for a balanced three-phase system, it is possible to remove the neutral wire and represent a balanced three-phase system by a three-wire circuit, as shown in *figure 8.10*.

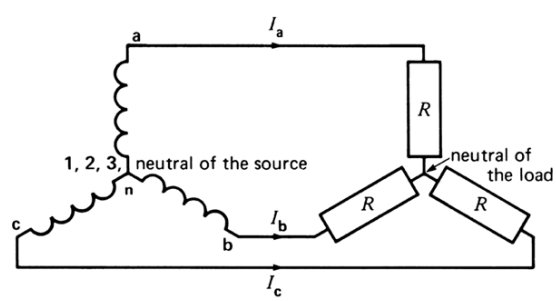


Figure 8.10: Three-Wire Three-Phase Balanced System

The three-phase system and the load shown in *figures 8.9* and *8.10* are said to be organized in *wye* configuration. The three-phase source is always in *wye*, whereas load can be connected either in *wye*, between line and neutral, or *delta*, between two lines (line-to-line).

8.5 Relationship Between Line-to-Neutral and Line-to-Line Voltages

Line-to-neutral voltage, also called *phase* voltage is between each phase and neutral. A load connected in *wye* always experience phase voltage across it (if there is no line impedance). Line-to-line voltage, also called simply *line* voltage is between two phases. Hence, line voltages in a three-phase system are $\mathbf{E}_{ab}$, $\mathbf{E}_{bc}$, and $\mathbf{E}_{ca}$. Let's look at the value of each line voltage.

$$\begin{aligned}\mathbf{E}_{ab} &= \mathbf{E}_{an} - \mathbf{E}_{bn} = E_p\angle 0^\circ - E_p\angle -120^\circ = E_p(1-(\cos(-120^\circ)+j\sin(-120^\circ)))\\ &= E_p\left(1+0.5+j0.866\right) = E_p\left(1.5+j0.866\right) = 1.732E_p\angle 30^\circ = \sqrt{3}E_p\angle 30^\circ \qquad (8.7)\end{aligned}$$

Similarly, the value of $\mathbf{E}_{bc}$ and $\mathbf{E}_{ca}$ can be given by,

$$\mathbf{E}_{bc} = \sqrt{3}E_p\angle -90^\circ \;\&\; \mathbf{E}_{ca} = \sqrt{3}E_p\angle 150^\circ \qquad (8.8)$$

Hence, the magnitude of line voltages is $\sqrt{3}$ times the magnitude of phase voltages:

$$E_{LL} = \sqrt{3}E_{LN} \qquad (8.9)$$

Also, line voltages are 120° apart, similar to phase voltages. A phasor diagram with phase and line voltages is shown in *figure 8.11*.

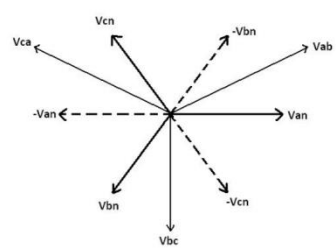


Figure 8.11: Phasor diagram of phase and line voltages

Example 8.2

A 3-phase generator generates a line voltage of 300V(peak). There is a balanced load of 100Ω connected to the system (refer to figure 8.9). Calculate (i) Voltage across each resistor (ii) Current in each resistor (iii) Total output power of the generator.

(i) Since load is connected in wye, voltage across each resistor is the line-to-neutral voltage;
→E_{LN}*(peak)* = E_{LL}*(peak)/sqrt*(3) = 173.21V

(ii) I_R = E_{LN}*(peak) /R* = 1.7321A

(iii) P= 3*E_{LN}*(peak)* *I/2 = 450W

8.6 Delta Connection

If a three-phase load is connected between line-to-line instead of line-to-neutral, it makes a delta connection, as shown in *figure 8.12*.

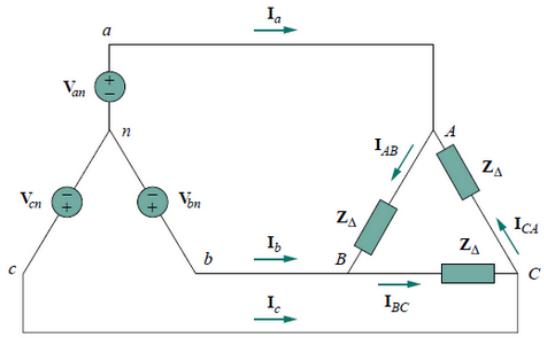


Figure 8.12: A Three-Phase load connected in delta

Hence, line-to-line voltage appears across each impedance of a delta-connected load. Let's look at the relationship between *line* currents (I_A, I_B, and I_C) and *load* currents (I_{AB}, I_{BC}, and I_{CA}):

$$\mathbf{I}_A = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \frac{\mathbf{E}_{AB}}{Z} - \frac{\mathbf{E}_{CA}}{Z} = \frac{E_p}{Z}\left[1.5 + j0.866 - (-1.5 + j0.866)\right] = \frac{3E_p}{Z}$$
$$= \frac{\sqrt{3}\sqrt{3}E_p}{Z} = \frac{\sqrt{3}E_{LL}}{Z} = \sqrt{3}I_{Load} \tag{8.10}$$

Likewise, $\mathbf{I}_B$ and $\mathbf{I}_C$ are given by,

$$\mathbf{I}_B = \sqrt{3}I_{Load}\angle -120^\circ \ \& \ \mathbf{I}_C = \sqrt{3}I_{Load}\angle 120^\circ \tag{8.11}$$

Hence, the magnitude of line currents is equal to $\sqrt{3}$ times the magnitude of load currents when the load is connected in delta configuration.

Example 8.3

A 3-phase system has a phase voltage of 220V(rms). There is a balanced load connected in delta to the system (refer to figure 8.12). Each line current is 10A. Calculate (i) load current (ii) magnitude of each impedance.

(i) $I_{LOAD} = I_{Line}/sqrt(3) =$ 5.7735A

(ii) Since load is connected in delta, voltage across each impedance is the line voltage;
→$E_{LL}(rms) = sqrt(3)*E_{LN}(rms) =$ 381.05V

Magnitude of each impedance: $Z = E_{LL}/I_{load} = 66\Omega$

8.7 Power Transmitted

Power transmitted by a three-phase system is three times the power transmitted by a single phase for a balanced load. Hence, different power quantities are given as follows:

- Active Power: $P = 3V_{LN}I_L\cos(\theta-\phi)$ W (8.12)
- Reactive Power: $Q = 3V_{LN}I_L\sin(\theta-\phi)$ VAR (8.13)
- Apparent Power: $S = 3V_{LN}I_L$ VA (8.14)
- Complex Power: $P+jQ$ VA (8.15)

Note that all voltage and current quantities in power equations are assumed to be RMS.

Example 8.4

A 3-phase 60Hz system has a line voltage of 600V. Three identical capacitors are connected in Δ to the system. Given that the line current is 20A, determine the capacitance value.

Load current: $I_{load} = I_{line}/sqrt(3) =$ 11.547A

Magnitude of the capacitive reactance: $Xc = E_{LL}/I_{load} =$ 51.962Ω

Capacitance: C= 1/(120*pi*Xc) = 51μF

Example 8.5

A 3-phase load draws 420KVA from a 3KV line voltage (line-to-line). Power factor of the load is 90% lagging. Calculate impedance of the load and draw phasor diagram of the system.

Since, it is not mentioned in which configuration the load is connected, assume it is connected in wye (if not given, always assume wye).

Apparent power per phase: $|S|_{per\ phase} = |S|_{total}/3 =$ 140KVA

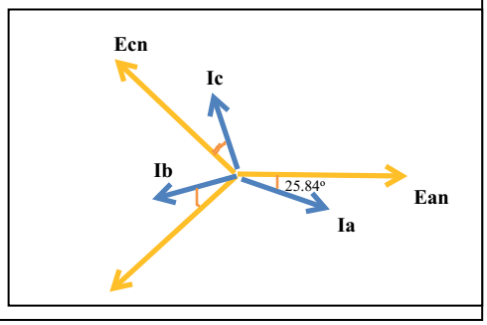
Line to neutral (phase) voltage: $E_{LN} = E_{LL}/sqrt(3) =$ 1732.1V

Load current: $I = |S|_{per\ phase} / E_{LN} =$ 80.829A

Magnitude of the load impedance: $Z = E_{LN}/I =$ 21.429Ω

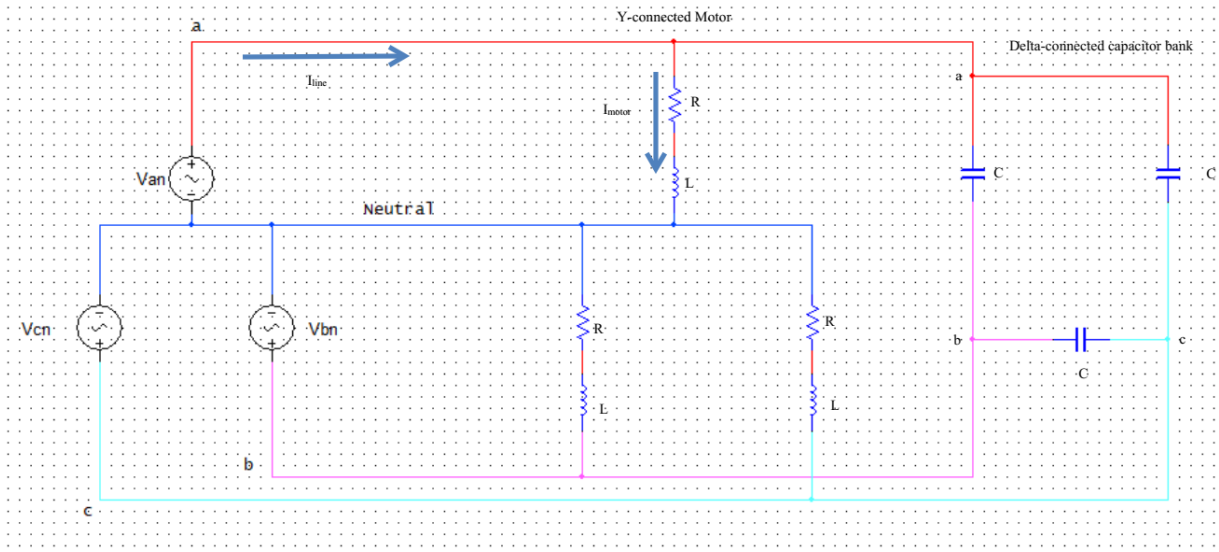
Impedance phase: $\angle Z = \cos^{-1}(0.9) =$ 25.842°

$R = Zcos(\angle Z) = 19.2857\Omega$; $X_L = Zcos(\angle Z) = 9.3405\Omega$



Example 8.6

A 3-phase Y-connected motor is rated at 1000hp. Line voltage is 4KV @ 60Hz. A Δ-connected capacitor bank is also connected to the same line, which is rated at 150KVAR. Output of the motor is 960hp and efficiency is 90%. Power factor of the motor is 88% lagging. Calculate (i) Active power absorbed by the motor (ii) Reactive power absorbed by the motor (iii) Reactive power supplied by the line (iv) Apparent power of the line (v) Transmission line current (vi) Motor current



>>Line-to-neutral voltage: $E_{LN} = E_{LL}/sqrt(3)$ = 2309.4V

(i) Input power of the motor: $Pm(in) = Pm(out)/\eta$ = 1066.7hp = 795.413KW

(ii) Apparent power of the motor: $Sm = Pm(in)/PF$ = 903.878KVA

>> Reactive power of the motor: $Qm = Sm*\sin(\text{acos}(0.88))$ = 429.318KVAR

(iii) Reactive power supplied by the line: $Q_{line} = Qm+Qc$ = 429.318K – 150K = 279.318KVAR

(iv) Apparent line power: S_{line}: $sqrt(Pm(in)^2 + Q_{line}^2)$ = 843.031KVA

Note: active power absorbed by the motor is the only active power in the system, which is supplied by the line.

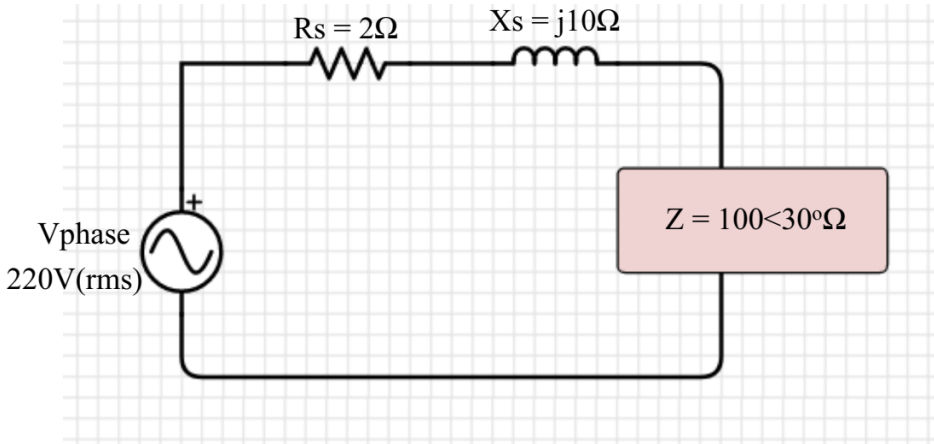
(v) Line current: $I_{line} = (S_{line}/3)/E_{LN}$ = 121.68A

(vi)Motor current: $Im = (Pm(in)/3)/(E_{LN}*PF)$ = 130.46A

Example 8.7

A 3-phase balanced load comprised of three impedances Z = 100∠30°Ω is connected to 60Hz, 220V(rms)(phase voltage) system. Synchronous (line) reactance and resistance are 10Ω and 2Ω per phase, respectively. Determine total active and reactive power delivered to the load.

Since the system is balanced, you can solve for a single phase and then extend your results for all the other phases. Single-phase equivalent circuit is shown in the following circuit diagram.



>> *Vphase* = 220V; *Rs* = 2Ω; *Xs* = j10Ω;

>> *Z* = 100*cos(30) + i*100*sin(30) = 86.603 + j50.000Ω

Line current: >> *I = Vphase/(Z + Rs + Xs)* = 1.7023 - j1.1528A = 2.0559∠-34.105°A

Active load power per phase: *P(per phase)* = $|I|^2 * R_{load}$ = 366.06W

Reactive load power per phase: *Q(per phase)* = $|I|^2 * |X_{load}|$ = 211.35VAR

Total load active power = 3**P(per phase)* = 1098.2W

Total load reactive power = 3* *Q(per phase)* = 634.04VAR

PROBLEMS

Note: If a 3-phase connection is not mentioned, always assume wye. If peak value is not mentioned then assume that RMS value of voltages and currents are given.

1. A large 3-phase, 4000V (line-to-line), 60Hz motor draws a current of 385A (per phase) and a total active power of 2344KW when operating at full load. Total copper losses are 15KW, iron losses are 23.4KW, and windage and friction losses are 12KW. Motor is rotating at 800 rpm. Calculate the following:
   (i) The power factor at full-load
   (ii) Output mechanical power
   (iii) Output torque
   (iv) Efficiency

2. A 3-phase generator possesses a synchronous reactance of 6Ω (line reactance between the source and the load) and the excitation voltage E_o = 3KV (source) per phase. Calculate the line-to-neutral voltage E_{Load} for a balanced resistive load of 8Ω. Also, draw a phasor diagram between three-phase source and load voltages.

3. A 3-phase generator connected in wye has the following characteristics: Eo = 2440V, line reactance Xs = 144Ω, line resistance Rs = 17Ω. Assume that there is 175Ω resistance connected as a load per phase. Calculate:
   (i) The line current
   (ii) Line-to-neutral and line-to-line voltage across the load
   (iii) Phasor diagram between source and load voltages

4. A 3-phase generator delivers power to a 2400 KVA (total), 16KV (line-to-neutral) load having a lagging power factor of 0.8. If the synchronous reactance (line reactance) is 100Ω per phase, calculate the magnitude of the induced voltage (source)Eo per phase.

5. Three 10W resistors are connected in delta on 208V (line-to-line), 3-phase line.
   (a) What is the value of each resistor?
   (b) If the fuse in one of the lines burns out, calculate the new total power supplied to the load (*Hint: circuit diagram of the system will help).*

6. An electric motor having PF of 82% lagging is connected to 600V , 3-phase line (line-to-line) and draws 25A,
   (a) Calculate the total active power supplied to the motor
   (b) If motor has 85% efficiency, calculate the mechanical output power.
   (c) How much energy does the motor consume in 3 hours (in Joule)?

7. Three identical impedances are connected in delta across a 3-phase , 600V line. If the line current in 10A and power factor is 70% lagging, calculate,
   (a) Impedance current
   (b) Impedance value (*R* and *X*)
   (c) Value of reactive component (*C* or *L)*

8. A three-phase balanced load is connected in **delta** configuration to a 311.13V (peak), 50Hz line-to-neutral three-phase system. The load current is 10A (peak) at 70% lagging power factor. Draw the equivalent system diagram and calculate:
   (i) Load impedance.
   (ii) Value of the inductor or value of the capacitor, whichever you think is part of the load.
   (iii) Magnitude of the line current.
   (iv) **Total** active and reactive power supplied by the three-phase system.

9. A three-phase balanced motor represented by a 50Ω inductive reactance in series with a 200Ω resistance is connected in delta configuration to a 60Hz, 120V(rms, LN) system. A balanced capacitor bank comprised of three 2μF is also connected to the 3-phase system in wye configuration. Determine:
   (i) Total active and reactive power received by the motor.
   (ii) Reactive power of the capacitor bank.
   (iii) Total active and reactive power supplied by the three-phase system.
   (iv) Power factor of the three-phase system.

10. A balanced load is connected in wye configuration to a 3-phase system with peak line-to-line voltage of 269.44V. Line frequency is 60Hz. Line resistance is 2Ω and reactance is 4Ω (per phase). The balanced load is comprised of 10Ω resistance in parallel with a 100μF capacitor per phase. Calculate:
    (i) Load voltage.
    (ii) Total Active and reactive power of load.
    (iii) Total active and reactive power of source.

Chapter 9 – The Ideal Transformer

Transformers are one of the most important devices used in power systems. The physical construction of a transformer is very simple: two coils separated by some distance wrapped on a ferromagnetic or insulating material, called *core*. *Figure 9.*1 shows the basic construction of an *iron-core* transformer and its electrical symbol.

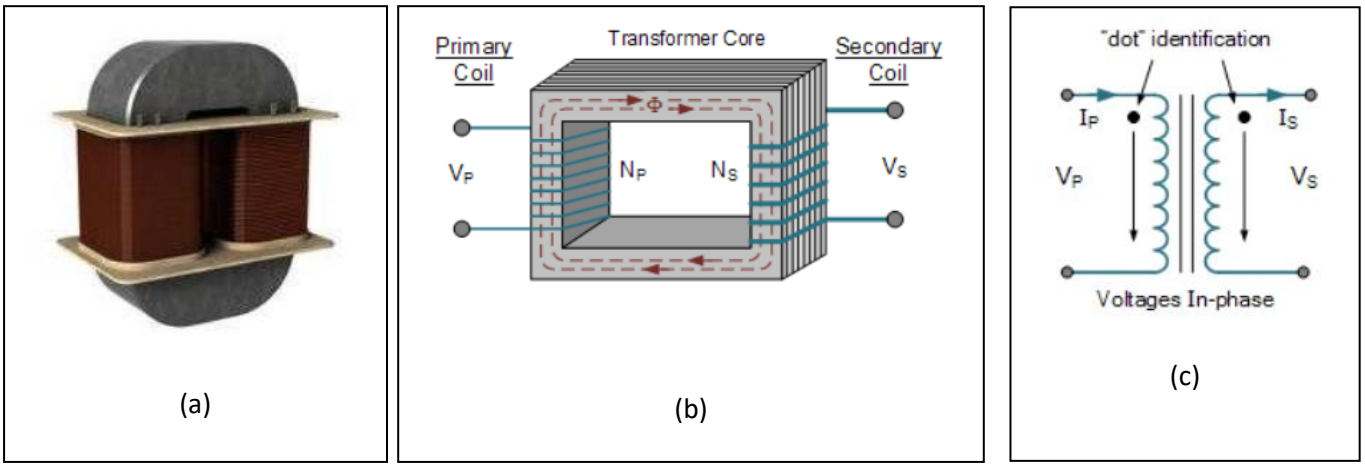


Figure 9.1: (a) Physical construction of an iron-core transformer

(b) Equivalent block diagram (c) Circuit representation

In power systems, generally, iron-core transformers are used. There are also *air*-core transformers, where core is comprised of an insulating material and the two sides are connected by the mutual flux through the air. These types of transformers are used in radio frequency transfer. *Figure 9.*2 shows the flux linkage of the two coils in an air-core transformer.

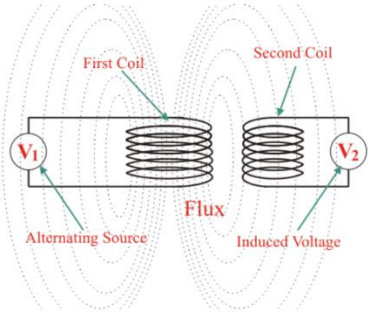


Figure 9.2: Air-core transformer

The two sides of a transformer are called *primary* and *secondary*. The primary side is usually connected to a power source and load is usually connected to the secondary side. Core of an iron-core transformer is usually constructed of highly permeable material made from thin silicon steel laminations, as shown in *figure 9.1(b)*. The highly permeable core can easily be magnetized and provides a path for the magnetic flux to move around the core to link both primary and secondary sides. Laminated plates reduce the effect of eddy currents and thus reduce power loss and heating of the core.

Some of the most important applications of transformers are:

- Step-up/Step-down voltages and currents: The most common use of transformers is to step-up/step-down voltages and currents from its primary to secondary side.
- Transformers are also used to step-up/step-down the apparent values of capacitors, inductors and resistors as seen by the source. This will be discussed a little later under one of the sections in this chapter.
- Circuit isolation (isolation transformers): Transformers are used to isolate load circuit from the source circuit to eliminate any DC input and suppress electrical shocks to the load.

This chapter deals with the study of ideal transformers. Concepts developed in the study of ideal transformers also work in the study of practical transformers. Ideal transformers differ from the practical transformers in few respects:

- All the flux in the core and in the die-electric between the two coils is linked with both primary and secondary coils equally. This flux is called *mutual flux*. There is also a flux, called *leakage flux*, which links only with one of the coils. It is assumed that the leakage flux is zero in ideal transformers.
- It is assumed that there are no hysteresis and eddy current losses in the core of an ideal transformer.
- It is assumed that the two coils have no resistance; hence, there are no copper losses in the primary and secondary coils.

To understand the operation of an ideal transformer, we will start with the discussion of voltage induced in a coil.

9.1 <u>Voltage Induced in a Coil</u>

According to Faraday's law of electromagnetic induction, when a time-varying flux is linked with a coil, a voltage is induced in that coil. If the time-varying flux is sinusoidal, it can be given by (9.1):

$$\phi(t) = \phi_p \cos(\omega t) \tag{9.1}$$

where ϕ_p is the peak value of the flux. The expression of the induced voltage in the coil can be derived using Faraday's law:

$$e_{ind} = -N\frac{d(\phi_p \cos(\omega t))}{dt} = N\omega\phi_p \sin(\omega t) \tag{9.2}$$

Since, this is a sinusoidal quantity, the root-mean-square (RMS) value of the induced voltage can be calculated by,

$$e_{ind}(rms) = \frac{N\omega\phi_p}{\sqrt{2}} = \frac{2\pi fN\phi_p}{\sqrt{2}} = 4.44\, fN\phi_p \tag{9.3}$$

9.2 Applied and Induced Voltages

When a coil is connected to a sinusoidal voltage source, a sinusoidal current start flowing through it. This current produces a magnetomotive force (mmf) in the coil (*NI*), which in turn produces flux. Since the current that is responsible to produce mmf is sinusoidal, the resultant flux is also sinusoidal. Since this flux is changing with respect to time, it will induce a voltage in the coil, according to Faraday's law of electromagnetic induction, as discussed in the previous section. This arrangement is shown in *figure 9.3.*

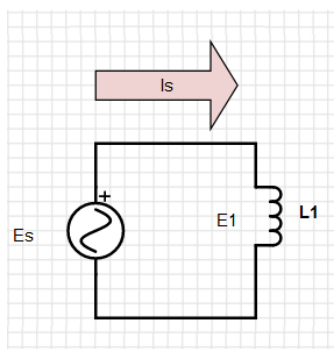


Figure 9.3: An inductor connected to a sinusoidal source

As discussed earlier, the rms value of the induced voltage can be given by (9.3), which in this case will be equal to the applied source voltage. Since current *Is* is flowing through a pure inductor, it will be lagging *Es* and E_1 by 90°, so does the flux produced by the current.

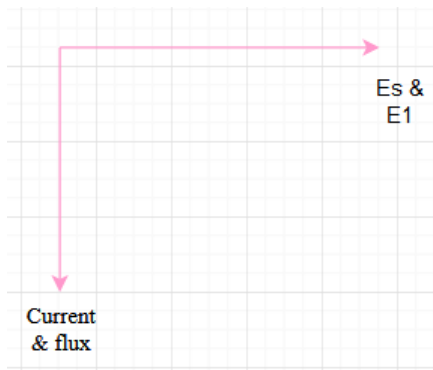
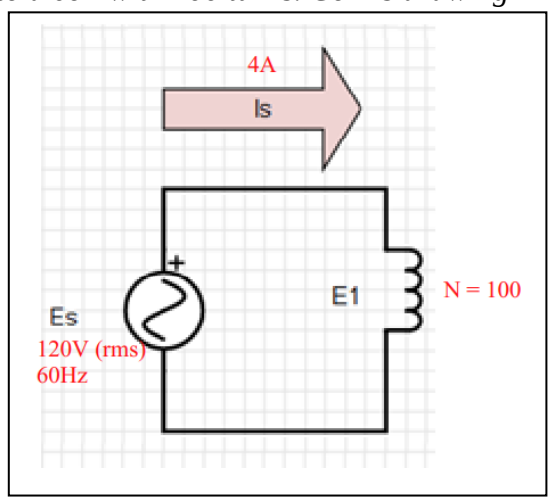


Figure 9.4: Phasor diagram for currents and voltages

<u>Example # 9.1</u>

A 120V (rms), 60 Hz sinusoidal source is connected to a coil with 100 turns. Coil is drawing 4A current. Determine:



(i) Maximum flux produced by the coil

(ii) Peak magneto-motive force (mmf)

(iii) Reactance of the coil

(iv) Inductance of the coil

<u>Solution:</u>

>> Es = 120V; f = 60Hz; N = 100; Is = 4A;

>> (i) Peak flux: $\phi = Es/(4.44*f*N)$ = 4.5045mWb

>> (ii) $mmf = N*Is_{peak}$

>> $mmf = N*sqrt(2)*Is$ = 565.69 A.t

>> (iii) Inductive reactance: $XL = Es/Is$ = 30Ω

>> (iv) Inductance: $L = XL/(2*pi*f)$ = 79.577 mH

9.3 Basic Principle of Transformers

Observe a transformer as shown in *figure 9.5*, where the primary side is connected to a sinusoidal supply and there is no load on the secondary side.

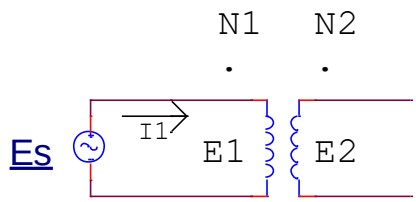


Figure 9.5: An ideal transformer without load

As discussed earlier, when a current flows through the primary coil with number of turns N_1, it will produce an mmf (N_1I_1), which in turn will produce a sinusoidal flux, which will induce a voltage E_1 in the coil. The flux will be linked to the secondary coil as well and induce a voltage E_2 in the secondary coil. Both voltages can be given using (9.3) as follows:

$$E_1 = 4.44\, fN_1\phi_p$$
$$E_2 = 4.44\, fN_2\phi_p \tag{9.4}$$

The ratio of the primary to secondary induced voltages will yield the following relationship:

$$\frac{E_1}{E_2} = \frac{4.44\, fN_1\phi_p}{4.44\, fN_2\phi_p} = \frac{N_1}{N_2} = a \tag{9.5}$$

Hence, the ratio between the induced primary to secondary voltages is equal to the ratio between the primary and secondary number of turns. This quantity is of vital importance in transformers, and it is called *transformer turn ratio* or simply *turn ratio*, generally represented by *a*.

Observe the two dots on the primary and secondary windings in *figure 9.1(c)* and *figure 9.5*. These dots represent that induced voltages on these ends are in-phase with each other.

From (9.5), it can be concluded that if number of turns on the primary side is more than that on the secondary side, voltage induced in the primary coil will be larger than the secondary coil and vice-versa. Hence, if $N_1 > N_2$, it is called a *step-down* transformer since secondary voltage will be less than the primary voltage. If $N_2 > N_1$, it will be a *step-up* transformer, with secondary voltage greater than the primary voltage.

9.4 Transformer under Load

When a load is connected to the secondary side of a transformer, induced voltage on the secondary side starts driving a current to the load, as shown in *figure 9.6.*

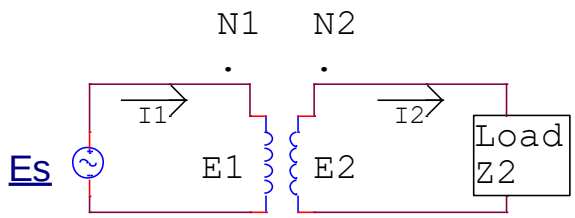


Figure 9.6: Transformer under load

As discussed earlier, in an ideal transformer, it is assumed that the flux linked with the primary and secondary coils is same (mutual flux); hence, the mmf of the two coils is also the same. Therefore, $N_1I_1 = N_2I_2$, which leads to the *current ratio*:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{9.6}$$

Therefore, the primary to secondary current ratio is the inverse of the transformer turn ratio.

From (9.5) and (9.6), one can write, $\frac{E_1}{E_2} = \frac{I_2}{I_1}$, which leads to an extremely important property of transformers:

$$E_1I_1 = E_2I_2 \tag{9.7}$$

From (9.7), it can be seen that the apparent power on both sides of a transformer is the same. Hence, a transformer does not change the apparent power. This means that for a step-down transformer, since $E_1 > E_2$, then $I_2 > I_1$, and vice-versa for a step-up transformer.

The load connected on the secondary side of the transformer can be given, using Ohm's law, by $Z_2 = E_2/I_2$. Since both sides are isolated from each other, the load that the source sees on the primary side can be calculated by $Z_1 = E_s/I_1$ or $Z_1 = E_1/I_1$. From the expressions of Z_1 and Z_2, one can find *impedance ratio* as follows:

$$Z_1 = \frac{E_1}{I_1} = \frac{aE_2}{\frac{I_2}{a}} = a^2\frac{E_2}{I_2} = a^2Z_2 \tag{9.8}$$

Therefore, the impedance that source sees on the primary side is square of the turn ratio times the actual impedance connected on the secondary side. Thus, a transformer has this property of increasing or decreasing the actual impedance value as seen by the source.

Example # 9.2

An ideal transformer is connected to a 220V, 60Hz sinusoidal source. There are 100 turns on the primary side and 3000 turns on the secondary side. A load is connected on the secondary side which is drawing 2A current at a power factor of 80% lagging. Determine:

(i) Primary current

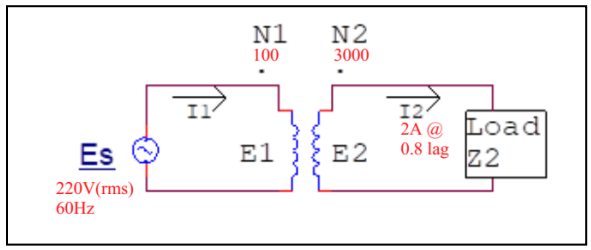
(ii) Secondary voltage

(iii) Load impedance

(iv) Individual components of the load

(v) Peak flux linked by the secondary winding

(vi) Phasor diagram of the system



Solution:

>> $Es = E1 = 220V$; $f = 60Hz$; $N1 = 100$; $N2 = 3000$; $I2 = 2A$; $PF = 0.8$;

(i) Primary current: $I1 = N2*I2/N1 = 60A$

(ii) Secondary Voltage: $E2 = E1*N2/N1 = 6600V$

(iii) Load impedance: $|ZL| = E2/I2 = 3300\Omega$

>> $\angle ZL = \cos^{-1}(0.8) = 36.870^{o}$

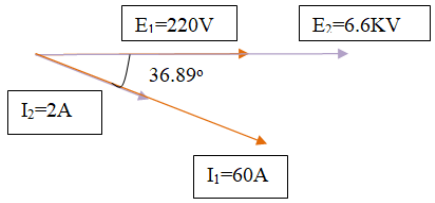
>> $ZL = |ZL| \cos(\angle ZL) + |ZL| \sin(\angle ZL)i = 2640.0 + 1980.0i\Omega$

(iv) $RL = \text{real}(ZL) = 2640\Omega$; $XL = \text{imag}(ZL) = 1980.0\Omega$

>> $L = XL/(2*pi*f) = 5.2521H$

(v) Peak flux linked with the secondary winding: $\phi = E2/(4.44*N2*f) = 8.2583mWb$

(vi) Phasor diagram of the system:



9.5 Impedance Shift

To emphasize on the important property of transformers that was discussed towards the end of the last section, let's reiterate it: the load impedance that the source on the primary side sees is larger than the actual value for a step-down transformer ($a > 1$) and smaller than the actual value for a step-up transformer ($a < 1$). The relationship between the voltage, current and impedance ratios as given by (9.5), (9.6), and (9.8) enables us to create a single-loop circuit of a transformer by either shifting secondary quantities to the primary side or primary quantities to the secondary side. This is generally referred to as *impedance shift*. Note that this impedance shifting is necessary in case if primary and secondary currents (I_1 *and* I_2) and induced voltage values (E_1 *and* E_2) are unknown. If any of those values are known, then the circuit can be solved without impedance shift.

Secondary side shifted to the primary side:

Observe a transformer circuit with primary and secondary line impedances, Z_1 and Z_2, and a load impedance Z_L, as shown in *figure 9.7*.

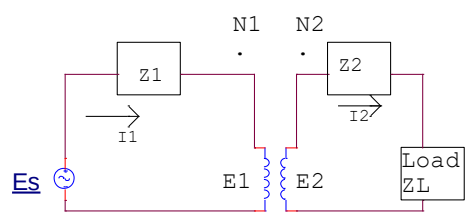


Figure 9.7: An ideal transformer with line impedances and load impedance

To eliminate the transformer, the secondary side can be shifted to the primary side with new impedance values calculated according to the relationship given in (9.8). This will yield a circuit with a single loop and calculation of different quantities can be done simply by using Ohm's law. The circuit corresponding to the secondary side shifted to the primary side is shown in *figure 9.8*.

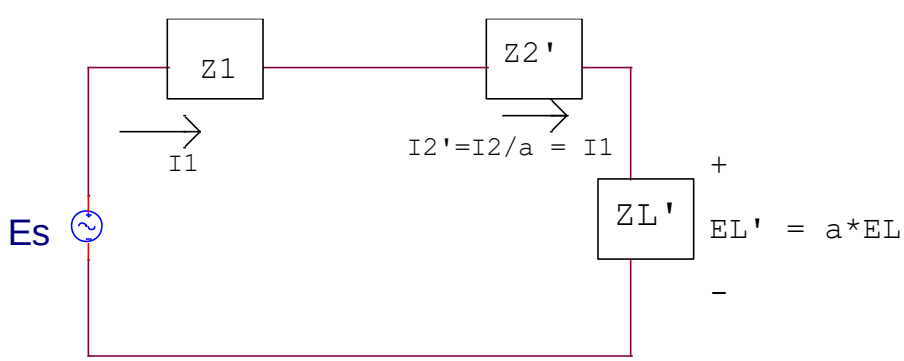


Figure 9.8: Secondary side reflected on the primary side

Using (9.5), (9.6), and (9.8), the reflected values on the primary side, shown by *prime (')* in *figure 9.8*, can be calculated as follows:

$$\begin{aligned} Z_2' &= a^2 Z_2 \\ Z_L' &= a^2 Z_L \\ I_2' &= I_2 / a \\ E_L' &= a E_L \end{aligned} \tag{9.9}$$

If most of the quantities required are primary side quantities (input power, primary current etc.), it will be convenient to shift secondary side to the primary and then solve the circuit.

Primary side shifted to the secondary side:

When primary side of the circuit from *figure 9.7* is shifted to the secondary side, again, (9.5), (9.6), and (9.8) are used to calculate the new shifted quantities, as shown by *double prime (')* in the reflected circuit shown in *figure 9.9*.

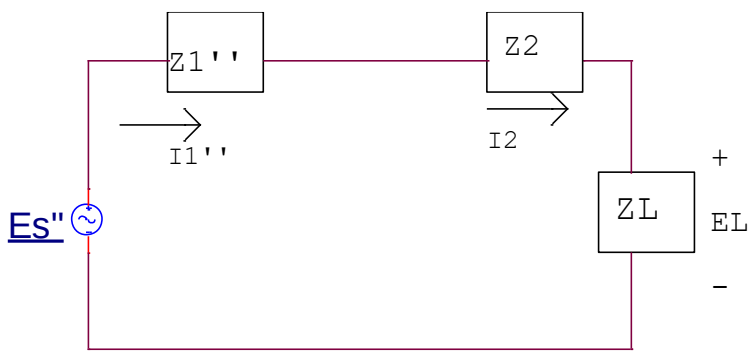


Figure 9.9: Primary side reflected on the secondary side

The reflected values can be calculated using (9.5), (9.6), and (9.8) as follows:

$$\begin{aligned} Z_1'' &= Z_1 / a^2 \\ I_1'' &= a I_1 \\ E_s'' &= E_s / a \end{aligned} \tag{9.10}$$

If most of the quantities required are on the secondary side (load power, load current and voltage etc.), it will be convenient to shift primary side to the secondary and then solve the circuit. This way, secondary side quantities will stay the same.

Note that since the power does not change on either side of the transformer, if load power is required, you can shift the circuit on the primary side and calculate the power in the reflected load value, which will be the same as the power in the actual load connected on the secondary side.

Example # 9.3

An ideal transformer is connected to a 110V, 60Hz source. Turn ratio is 10. Primary line reactance is 3Ω and resistance is 1Ω. Secondary line reactance is 0.02Ω and resistance is 0.005Ω. There is a capacitive load connected on the secondary with 0.1Ω resistance and 10mF capacitance. Determine:

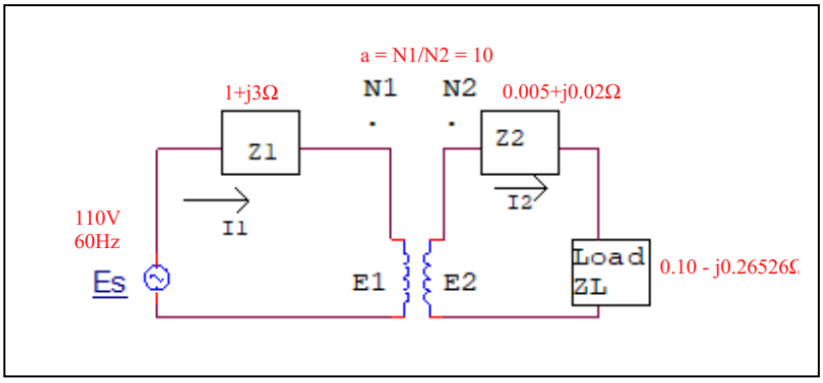
(i) Primary current

(ii) Secondary current

(iii) Load voltage

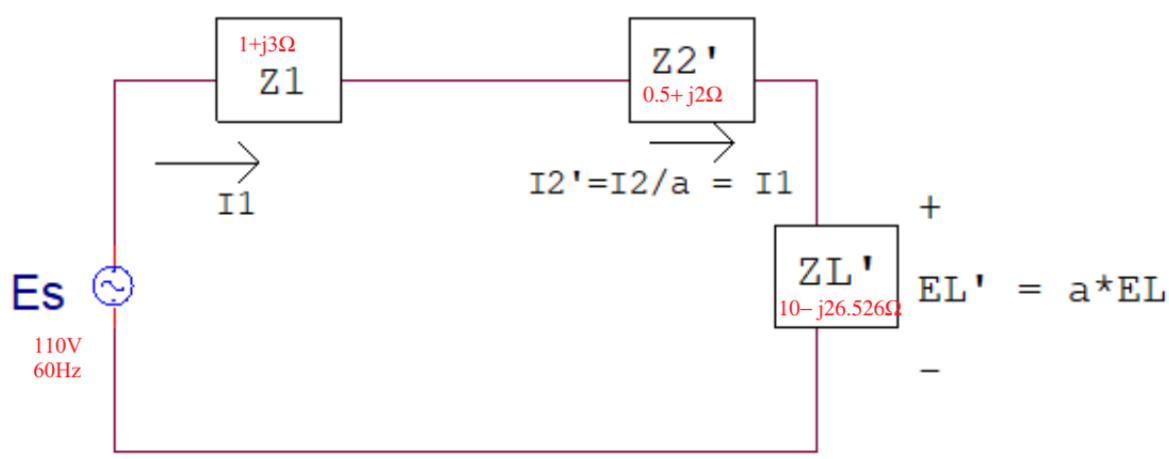
(iv) Active power of the load

(v) Reactive power of the load



Solution:

Since *I1*, *I2*, *E1*, and *E2*, all are unknown, we will have to reflect one side on another to solve this problem. Let's reflect the secondary side on the primary:



>> *Es* = 110V; *f* = 60Hz; *a* = 10; Z1 = 1+3iΩ; Z2 = 0.005+0.02iΩ; CL = 10mF;

>>ZL = 0.1 + 1/(*j2πfCL*)) = 0.10000 - 0.26526iΩ

Reflecting Z2 and ZL on the primary side:

>> Z2' = *a*^2*Z2 = 0.50000 + 2.00000iΩ

>> ZL' = *a*^2*ZL = 10.000 - 26.526iΩ

(i) Primary current: *I1 = Es/(Z1+Z2'+ZL')* = 2.1239 + 3.9755iA = 4.5072∠61.887°A

(ii) Secondary current: *I2 = a*I1* = 21.239 + 39.755iA = 45.072∠61.887°A

(iii) Load voltage: *EL = I2*ZL* = 12.6692 - 1.6583iV = 12.77∠-7.457°V

(iv) Complex load power: S = *EL*conj(I2)* = 203.15 - 538.88i VA

>> Active power: *P* = real(S) = 203.15W

(v) Reactive power: *Q* = imag(S) = -538.88VAR

PROBLEMS

1. An ideal transformer has 500 turns on the primary and 300 turns on the secondary. The source produces a voltage of 600V (RMS) and load *Z* is a pure resistance of 12Ω. Calculate:
   (i) E_2 (ii) I_2 (iii) I_1 (iv) Power delivered to the primary (W) (v) Power output from secondary (W)

2. In *problem 1*, what is the impedance seen by the source?

3. A coil with an air core has a resistance of 14.7Ω. When it is connected to a 42V, 60 Hz ac source, it draws a current of 1.24A. Calculate:
   (i) The impedance of the coil
   (ii) Reactance of the coil and its inductance
   (iii) Phase angle between the voltage and current

4. A 40μF, 600V paper capacitor is available but we need one having rating of about 300μF. It is proposed to use a transformer to modify the 40μF so that it appears as 300μF. The following transformer ratios are available: 120V/330V; 60V/450V; 480V/150V. Which transformer is the most appropriate and what is the reflected value of the 40μF capacitance? To which side of the transformer should the 40μF capacitor be connected?

5. An ideal transformer is connected to a 110V(rms), 60Hz source. Line impedance on the primary is 4+5iΩ and on the secondary side is 1+2iΩ. A 1mF capacitor is connected as load on the secondary side. Transformer has 100 turns on the primary side and 50 turns on the secondary side. Calculate:

   (i) Primary and secondary current
   (ii) Active and reactive power supplied by the source.
   (iii) Power factor of the source.

6. A single-phase ideal transformer is connected to 220V, 60Hz line. There are 1000 turns on the primary winding and 200 turns on the secondary winding. Load connected to secondary is drawing 2A current at 0.8 power factor lagging.
   (i) Draw a well-labeled circuit diagram.

(ii) Calculate primary current.
(iii) Calculate secondary voltage.
(iv) Calculate impedance value in both Cartesian and polar form.
(v) Calculate the value of inductor or capacitor in the load impedance.
(vi) Draw phasor diagram between primary and secondary voltages and currents.

7. A sinusoidal source $155.56\cos(120\pi t)$V is connected to the primary side of an ideal transformer. Transformer's turns ratio is 10. A load is connected on the secondary side which is drawing 1200W of real power. Power factor of the load is 80% lagging. Determine the complex power supplied by the source.

8. An ideal transformer has 100 turns on the primary side and 400 turns on the secondary side. It is connected to a 220V(rms), 50Hz source. Line resistance on the primary side is 4Ω and inductive reactance is 5Ω. Secondary line resistance is 32Ω and inductive reactance is 64Ω. A load comprised of 160Ω resistance in series with a 10μF capacitor is connected on the secondary side. Determine:

   (i) Total active and reactive power supplied by the source.
   (ii) Total active and reactive load power.
   (iii) Phasor load voltage.

Chapter 13 – Three-Phase Induction Machines

Three-phase induction machines are comprised of both three-phase induction motors and generators. We will concentrate on mainly three-phase induction motors in this chapter. Three-phase induction motors are AC motors with no DC power involved. They are widely used in industry due to their simplicity, ruggedness, low price, and easy maintenance.

The construction of a three-phase induction motor is simple: three-phase windings are connected to the *stator* that carry current to electromagnets to produce a three-phase magnetic field, and a *rotor* is placed within the stator that is acted upon by the Lorentz force to rotate around an axis. The difference between induction motors and DC motors that were discussed earlier is that the rotor (armature) is not connected to any power supply that drives current through it. Rather, current flows in the rotor due to the voltage that is induced in it due to the construction of rotor and its interaction with the three-phase flux, as will be explained later in this chapter. The operation of induction motors will be explained assuming the rotor to be a *squirrel-cage* rotor.

13.1 Force on a Conducting Ladder

Assume a ladder constructed of a conducting material where current can easily flow, as shown in *figure 13.1*. Observe that steps of the ladder are shorted out with the two sides of the ladder. Imagine a situation where you are holding a permanent magnet over the ladder and moving it along the ladder with some velocity '**v**', as shown in the figure. When the magnetic flux will link with the ladder, and it will change due to the motion of the magnet, a voltage will induce in the steps of the ladder in accordance with the Faraday's law of electromagnetic induction.

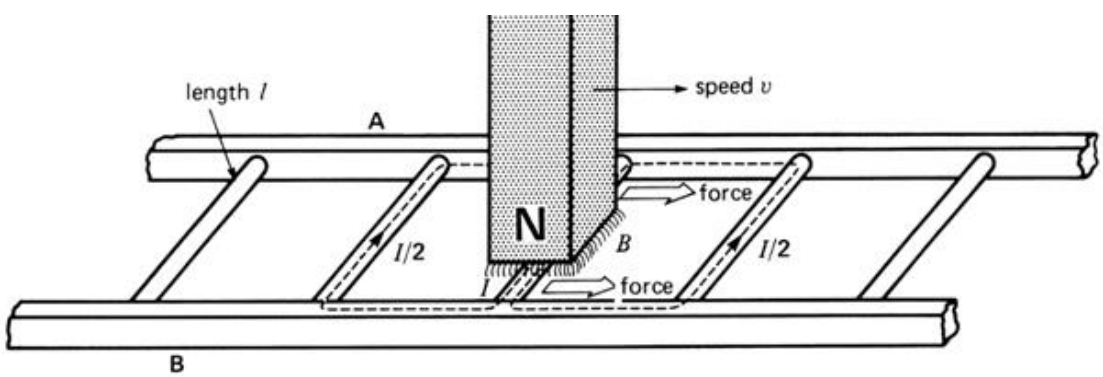


Figure 13.1: Interaction between a magnet and a conducting ladder

The direction of the induced voltage can be determined using the expression, e_{ind} = **l.(v x B)**. Assume that the current flow due to the induced voltage is in the positive-*x* direction. Hence, length vector will have a direction '**i**'. Since magnet is moving in the positive-*y* direction, the velocity vector of the moving magnet will have '**j**' direction. Remember, in the equation to calculate the induced voltage, velocity vector is that of the moving conductor, not moving magnet. Hence, the velocity of the conductor will be taken as *relative velocity* with respect to the magnet. When the conducting ladder is at rest, its magnitude of velocity will be the same as the magnitude of velocity of the moving magnet, however, direction will be opposite. Hence, the direction of the velocity of the conducting ladder will be in the '**-j**' direction. Magnetic flux density, **B**, is in the '**–k**' direction. Therefore, the direction of the induced voltage in the steps of the conducting ladder will be **i . (-j** x **–k)** = **i . i** = 1. Therefore, our assumed direction of the flow of current is correct, i.e., current will be coming out of the induced voltage in the positive-*x* direction, and voltage induced will be positive in the positive-*x* direction.

As shown in *figure 13.1*, when the voltage is induced in the conducting ladder, current starts flowing in it due to the short-circuit nature of the conducting ladder steps. Current will flow out of the step where voltage is induced and flow in the adjacent steps completing a loop. Now we have a conductor carrying current interacting with the magnetic field. What will be the result? Conducting ladder will start experiencing Lorentz force, **F** = I (**l** x **B)**. Since the direction of the length vector is '**i**' and the direction of the magnetic flux density vector is '**-k**', force on the ladder will be in the '**j**' direction. Hence, if ladder is free to move, it will start moving in the same direction as the moving magnet.

When ladder starts moving, the magnitude of the relative velocity between the moving magnet and the moving ladder, $|v_{\text{ladder}} - v_{\text{magnet}}|$, becomes smaller. Hence, the Lorentz force on the ladder also becomes smaller but since now ladder is already in motion, it does not require a large force to keep itself in motion (remember our discussion of a moving armature in a DC motor). The conducting ladder will keep on accelerating until its velocity becomes quite close to the velocity of the moving magnet, at which time, it will stop accelerating and starts moving with a constant velocity. The ladder can never catch up with the moving magnet, i.e., their velocities cannot become equal as there will always be some force required to overcome frictional forces and keep the ladder moving with a constant velocity. If the ladder does catch up with the moving magnet, the relative velocity will become zero and Lorentz force on the ladder will go down to zero, and the ladder will start decelerating. As soon it its velocity goes down, the relative velocity becomes greater than zero and Lorentz force will start acting on it again.

With all this discussion about how a conducting ladder can accelerate under the influence of a moving magnet, imagine if you bend the ladder upon itself to make a close loop, as shown in *figure 13.2*, and use this loop as the armature of a motor which is free to rotate about an axis. If this loop is acted upon by a moving magnetic field, it will start moving about the axis. This is the principle of three-phase AC induction motors, and the armature crafted in such a way is called a *squirrel-cage* rotor.

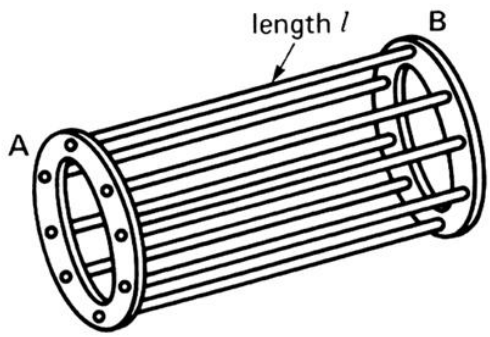


Figure 13.2: Squirrel-cage rotor

13.2 Rotating Magnetic Field

A three-phase induction motor has three-phase electromagnets on its *stator*. Each phase is connected to its corresponding electromagnet, as shown in *figure 13.2*.

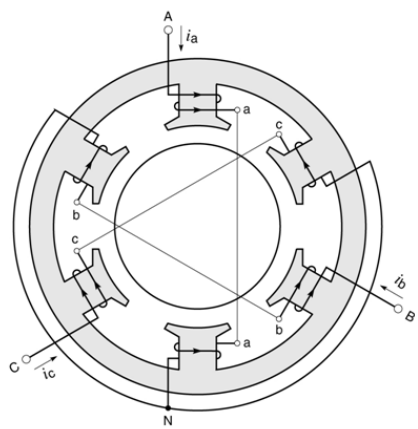


Figure 13.2: Three phases connected to three electromagnets (two poles per magnet)

As three-phase current flows in the windings of electromagnets, a three-phase flux is produced. Like three-phase current, this three-phase flux is also 120° apart from each other. At each instant of time, the resultant flux from the three phases adds up in such a way that it points to one direction. Over one input cycle (360°), the resultant flux also rotates around one cycle. In this way, a three-phase field produces a rotating magnetic field. This is shown in *figure 13.3.*

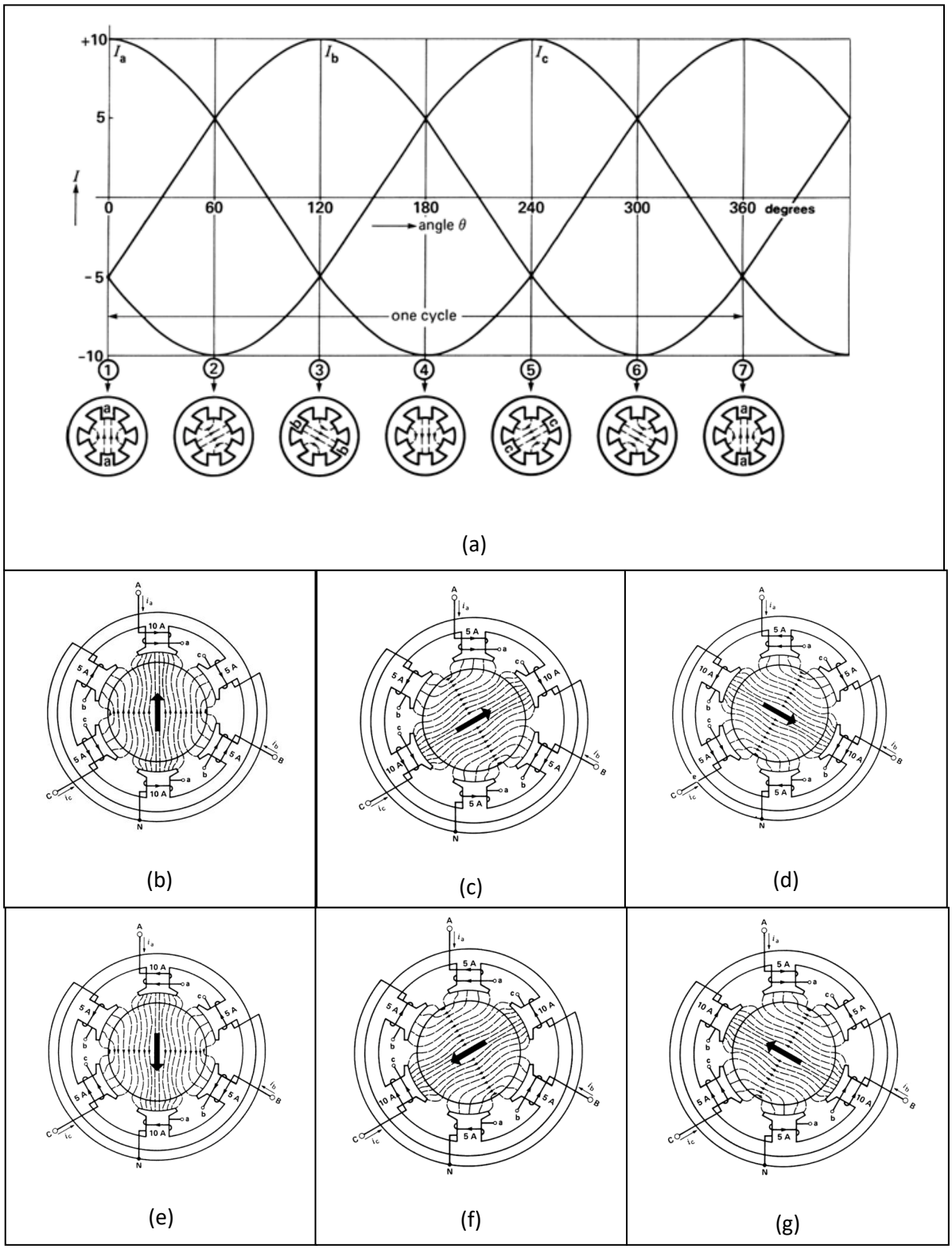


Figure 13.3: (a) Instantaneous values of three-phase currents (b)-(g) Resultant flux at instances 1 through 6 as shown in (a)

The speed with which the flux is rotating is called *synchronous speed*. Synchronous speed (n_s) depends on the input voltage frequency (f) and number of poles per phase (P). Mathematically, it is given by the following equation:

$$n_s = \frac{120f}{P} \qquad \text{(rpm)} \tag{13.1}$$

Hence, for two poles per phase, as shown in *figure 13.2*, and with an input frequency of 60Hz, the synchronous speed will be 3600rpm or 60Hz, i.e., one complete input cycle will produce one complete rotating cycle of the flux.

It is possible to create a group of poles per phase with more than two poles. This type of stator arrangement will produce a flux with synchronous speed less than the input frequency. For an *n*-group per phase stator, the groups are connected in series in such a way that when the phase current flows through them, it creates *n* alternating N-S poles. *Figure 13.4* shows a stator with four and eight groups of poles.

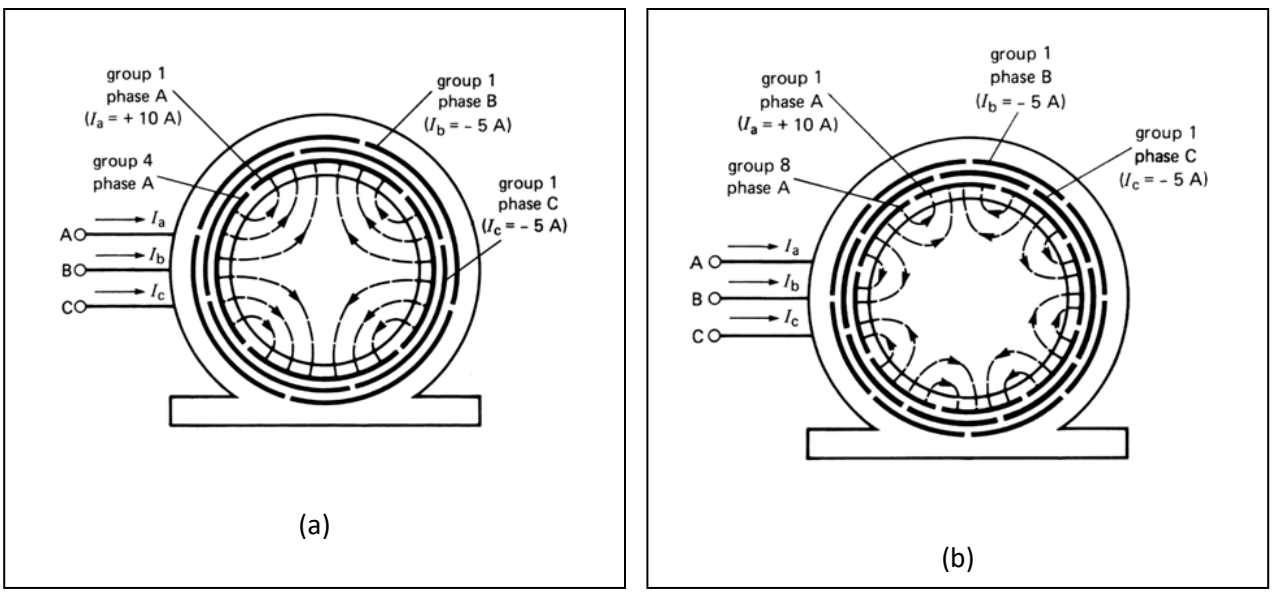


Figure 13.4: (a) Four poles per phase (b) Eight poles per phase

It is easy to change the direction of the revolving flux by changing the sequence of three phases connected to the electromagnet on the stator. If sequence *ABC* is producing a revolving flux in counterclockwise direction, changing the sequence to *ACB* will make the flux revolve in clockwise direction. This can be used to apply electrical braking as well as changing the direction of rotation of the rotor, as will be discussed next.

13.3 Working of a Squirrel-cage Three-Phase Induction Motor

The squirrel-cage rotor is placed in the center of the three-phase stator of the induction motor. The rotor is free to rotate about its axis. When the three-phase current produces a three-phase flux, the resultant of which revolves around with synchronous speed, it induces a voltage in the steps of the squirrel-cage rotor in a similar way as the moving magnet induces voltage in the conducting ladder. This induced voltage drives a current in the squirrel-cage rotor. The squirrel-cage rotor experiences Lorentz force due to the interaction between the revolving magnetic field and the current that is flowing through it, which produces a torque that starts moving the rotor in the same direction as the direction of the revolving magnetic field.

The initial voltage induced in the rotor when it is at rest is quite large, since the magnitude of the relative velocity of the rotor with respect to the revolving magnetic flux is maximum. This large voltage runs a large starting current in the rotor, due to which it experiences a large Lorentz force which creates a large torque to move the rotor rapidly from its resting position. Once it starts rotating, magnitude of the relative velocity becomes smaller, which in turns results in a smaller induced voltage, which runs smaller current in the rotor. The Lorentz force goes down, since the value of the current is smaller and hence, torque becomes smaller as well. But since the rotor is moving now, it does not require as large of a force and torque to accelerate. The rotor keeps on accelerating until it reaches very close to synchronous speed and then it starts rotating with a constant velocity. The difference between the synchronous speed (n_s) and the rotor speed (n) is called *slip speed*. There is also a parameter, called *slip*, which is used to measure the slip speed with respect to the synchronous speed, and it is given by,

$$s = \frac{n_s - n}{n_s} \tag{13.2}$$

At no load, slip is around 0.1% since rotor speed is practically equal to the synchronous speed.

When the motor is connected to a load, it starts slowing down. When rotor speed decreases, slip increases, which results in the increase of induced voltage in the rotor. When induced voltage is increased, short-circuit current in the rotor is increased, which results in a larger Lorentz force, and hence, larger torque. The speed of the rotor keeps on decreasing and rotor torque keeps on increasing as long as it becomes equal to the torque exerted by the load on the rotor. When the load torque becomes equal to the rotor torque, the motor starts rotating at a constant speed, which is a little less than the no-load speed. Under normal loads, induction motor runs very close to synchronous speed. At full load, the slip for large motors (1MW and more) rarely exceeds 0.5% and for smaller machines (10KW or less), it seldom exceeds 5%. That is why induction motors are considered to be *constant speed* machines. Also, since they never actually run at synchronous speed, sometimes they are also referred to as *asynchronous* machines.

Example 13.1

a 3-phase, 50Hz source excites an 8-pole induction motor. If the motor is running at full-load with a speed of 725rpm, calculate slip.

Solution:

>> f = 50Hz; Poles = 8; n = 725 rpm;

Synchronous Speed: $ns = 120*f/Poles =$ 750 rpm

Slip speed = $ns - n =$ 25 rpm

Slip: $s = slip\_speed/ns =$ 0.033333 or 3.3%

13.4 Voltage and Frequency Induced in the Rotor

The voltage and its frequency induced in the rotor bars, both are function of slip, and are given by,

$$\begin{aligned} E_{rotor} &= sE_{oc} \text{ (approx)} \\ f_{rotor} &= sf \end{aligned} \tag{13.3}$$

where E_{oc} is the open-circuit voltage induced in the rotor when it is at rest, and f is the frequency of the input phase voltage. Note that for the squirrel-cage motor, the open-circuit voltage in the rotor bars is measured when the bars are disconnected from the end rings. Also, note that the frequency of the induced voltage and current is always given by the equation shown in (13.3), however, the induced voltage is given by the equation only if the revolving flux remains absolutely constant. However, from no-load to full-load, the actual value of the induced rotor voltage is slightly less than the shown equation.

Example 13.2

A 3-phase, 8-pole induction motor is connected to 110V (LN, rms), 50Hz system. Calculate the frequency of the voltage induced in the rotor (i) If motor is rotating at 700 rpm in the same direction as the rotating field, and (ii) If motor is rotating at 500 rpm in the direction opposite to the rotation of the magnetic flux

Solution:

>> poles = 8; ELN = 110V; f = 50Hz; n1 = 700rpm; n2 = 500rpm;

Synchronous speed: *ns* = 120**f/poles* = 750 rpm

(i) Slip: *s = (ns-n1)/ns* = 0.066667

Frequency of the induced voltage: *fr = s*f* = 3.3333 Hz

(ii) Slip: *s = (ns-(-n2))/ns* = 1.667 (*If motor speed is in the opposite direction to* n_s*, take it as negative)*

Note: A slip > 1 represents that electrical braking is applied on the rotor and it is slowing down

Frequency of the induced voltage: *fr = s*f* = 83.333 Hz

Note: When motor is at rest (locked rotor), speed is 0 and slip is 1. Frequency of the induced voltage will then be equal to the line frequency.

13.5 Line Current under Different Conditions

At no load, the stator current (line current) is about 0.3 to 0.5 times the full-load current. This current is mainly required to produce magnetic flux that links with the rotor (reactive power), and to compensate for the stator winding loss and small frictional and windage losses in the rotor (active power). Since there is no output power at no-load, efficiency is zero. The power factor at no load is low; it ranges from 20% for small machines and 5% for large machines.

When a motor is loaded, rotor current increases and it produces its own magnetic field. This magnetic field interacts with the stator magnetic field and the mutual flux pattern changes a little. In addition to mutual flux, stator and rotor produce leakage flux as well. The stator current increases to compensate for the reactive power required for these three types of flux. In addition to a little increase in the reactive power, the active power is increased quite a bit to convert electrical power into mechanical to support the load. Hence, the stator current increases quite a bit to compensate for the increase of both reactive and active powers. At full load, the power factor of the motor ranges from 80% for smaller machines to 90% for larger machines. The efficiency of the motor is very high and can go up to 98% for larger machines.

When the rotor is locked (rotor is at rest), very large voltage is induced in the rotor bars and starting current is extremely large; 5 to 6 times the full-load current. This increases the rotor copper loss from 25 to 36 times the loss at full load (I^2R). The leakage flux of the stator and rotor also increases. Although the output power of the locked rotor is zero but the large copper loss in addition to the increase of reactive power require large line current. Due to the large power loss, the rotor should never remain locked for more than a few seconds.

The full-load current of a three-phase induction motor is approximately given by the following equation:

$$I = \frac{600P_h}{E_{LL}} \tag{13.4}$$

where P_h is the output horsepower of the motor and E_{LL} is line-to-line input voltage.

Example 13.3

If starting current (locked rotor) of an induction motor is five times the full-load current and no-load current is half of the full-load current of a 1000hp, 120V(phase) motor, calculate full-load, no-load, and locked rotor currents. Also calculate the apparent power drawn at full-load.

Solution:

>> Motor_hp = 1000; *(Make sure to convert it into hp if given in watt)*

>> *ELN* = 120V; *ELL* = sqrt(3)*120; *((13.4) calls for line-to-line voltage)*

Full-load current: I_{FL} = 600**Motor_hp/ELL* = 2886.8A or 2.886 KA

No-load current: $I_{NL} = 0.5*I_{FL}$ = 1443.4A or 1.443 KA

Locked-rotor current: $I_{Locked\_rotor} = 5*I_{FL}$ = 14433.75673 or 14.433KA

Apparent power at full-load: $S = 3*ELN*I_{FL}$ = 1039230.48454 VA or 1.039 MVA (*Assuming RMS values of currents and voltages)*

13.6 Power Flow in a Three-Phase Induction Motor

Three phase active power enters into the stator of the motor, which can be given by,

$$Pin = 3V_{LN}I_L\cos(\theta) \tag{13.5}$$

There are two losses in the stator: *stator copper loss* and *stator core loss*. Copper loss is due to the heat dissipated due to the resistance of the stator winding when current flows through it. Core loss, which is also called iron loss, is comprised of hysteresis and eddy current losses in the iron core. Copper loss is calculated through the usual I^2R equation:

$$P_{SCL} = 3I_L{}^2R_{stator} \tag{13.6}$$

The value of core loss is generally provided. At this point, the remaining power is transmitted to the rotor through the air gap between the rotor and the stator. Hence, it is called *air gap power* or simply *rotor power*,

$$P_r = P_{in} - P_{SCL} - P_{core} \tag{13.7}$$

The first loss that is considered in the rotor is *rotor copper loss*. This is calculated as a product of *slip* and the *rotor power*,

$$P_{RCL} = sP_r \tag{13.8}$$

where *slip* is calculated using (13.1) and (13.2). The power available after this point is called *induced* or *mechanical* power,

$$P_M = P_r - P_{RCL} = P_r - sP_r = (1-s)P_r \tag{13.9}$$

The next power loss is due to *friction* and *windage*, which is usually given. The remaining power is the *output* or *shaft* power available to move the load.

$$P_{out} = P_M - P_{F\&W} \tag{13.10}$$

If frictional and windage losses are negligible then mechanical power is equal to the output or shaft power. Also, if core, windage, and frictional losses are not given but one can calculate other losses and the output power then core, windage, and frictional losses can be calculated by subtracting other losses and the output power from the input power.

Efficiency of a motor is given by the ratio of the output to the input power, as discussed earlier in *section 6.4*. *Figure 13.5* shows the active power flow routine as discussed in this section.

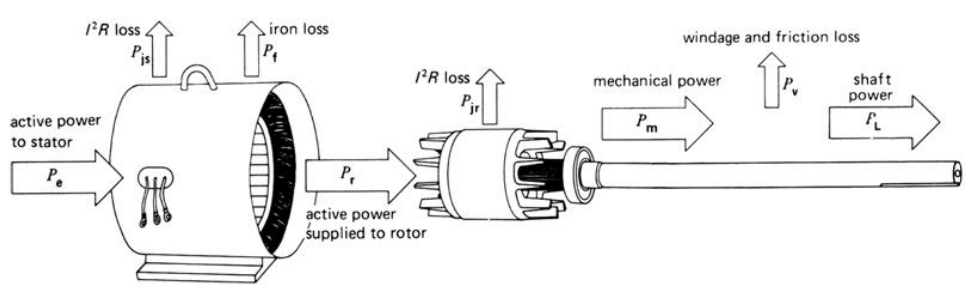


Figure 13.5: Power Flow in a Three-Phase Induction Motor

13.7 Motor Torque

Output torque of the motor is given by,

$$T = \frac{9.55P_{out}}{n} \tag{13.11}$$

If frictional and windage losses are negligible then output power and mechanical power will be approximately equal and torque of the motor will be given by,

$$T = \frac{9.55P_M}{n} = \frac{9.55(1-s)P_r}{n_s(1-s)} = \frac{9.55P_r}{n_s} \tag{13.12}$$

Remember, the unit of torque is Newton-meter (N.m)

13.8 Speed-Torque Relationship

The torque developed by a motor depends upon its speed but the relationship between the two is not very easy to express in a closed-form equation. Instead, *figure 13.6* shows the torque vs. speed relationship of a three-phase induction motor with nominal full-load torque of *T*.

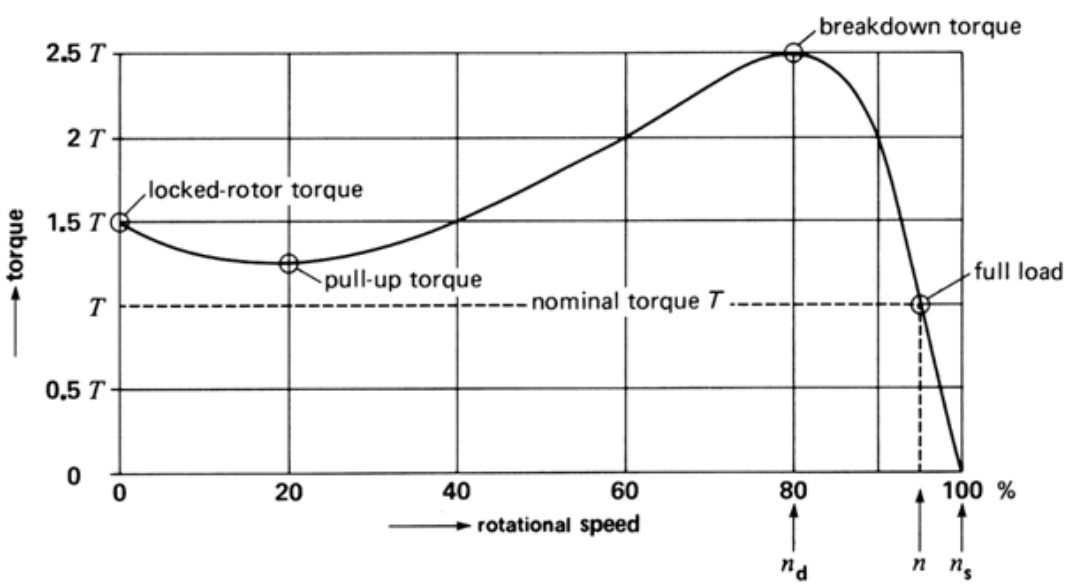


Figure 13.6: Speed-Torque Relationship of a 3-Phase Squirrel-Cage Motor

As shown in the figure, the starting torque is 1.5 times and the maximum torque, also called breakdown torque, is 2.5 times the nominal full-load torque. *Pull-up torque* is the minimum torque developed by the motor when it is accelerating from rest to the breakdown torque.

As discussed earlier, when a load is attached to a motor, it slows down, which results in a drop of the relative velocity, which increases the induced voltage and the rotor current, which in turns increases the Lorentz force on the rotor, which results in a greater torque developed by the rotor. The torque of the motor increases until it becomes equal to the torque required by load, at which point, motor, once again, starts spinning with a constant speed, which may be slightly less than the speed without load. If the connected load exerts a very high torque that exceeds the breakdown torque, the motor will quickly stop.

Smaller motors (15hp or less) develop their breakdown torque at a speed of about 80% of the synchronous speed (shown as n_d in *figure 13.6*). Large motors (1500hp or more) attain their breakdown torque at about 98% of the synchronous speed.

<u>*Example 13.4*</u>

A 3-phase, 4 KV (LL, rms), 60 Hz squirrel-cage induction motor draws a current of 385A and a total active power of 2344 KW when operating at full-load. It has 10 poles per phase. Speed of the rotor is 709.2 rpm. Stator resistance per phase is 0.05Ω. The total iron losses are 23.4KW and windage and friction losses are 12KW. Calculate:

(i) The power factor at full-load; (ii) The active power supplied to the rotor; (iii) Total copper losses in the rotor; (iv) Efficiency of the motor; (v) Output torque of the motor

<u>Solution:</u>

>> *ELL* = 4000V; *ELN = ELL/sqrt(3);* *f* = 60Hz; I_{FL} = 385A; P_{motor} = 2344KW; *poles* = 10; *n* = 709.2rpm; *Rs* = 0.05Ω; *iron_loss* = 23.4KW; *friction_loss* = 12KW;

(i) PF at full-load:

Apparent power: $S = 3*ELN*I_{FL}$ = 2667358.24366 VA = 2667.358KVA

Power factor: $PF = P_{motor}/S$ = 0.87877 or 87.877% lagging, since motor is mostly inductive.

(ii) The active power supplied to the rotor:

Stator copper loss: $Pscl = 3*I_{FL}^2*Rs$ = 22233.75000 W = 22.233KW

Rotor power: $Pr = P_{motor} - Pscl - iron\_loss$ = 2298366.25000W = 2298.366KW

(iii) Total copper losses in the rotor:

Synchronous speed: $ns = 120*f/poles =$ 720 rpm

Slip: $s = (ns-n)/ns =$ 0.015000

Rotor copper loss: $Prcl = s*Pr =$ 34475.49375W = 34.475KW

(iv) Efficiency of the motor:

Output power: $Pout = Pr - Prcl - friction\_loss =$ 2251890.75625W = 2251.890KW

Efficiency: $\eta = Pout/P_{motor} =$ 0.96070 or 96%

(v) Output torque of the motor:

Torque: $T = 9.55*Pout/n =$ 30323.68404 N.m = 30.323 KN.m

PROBLEMS

1. Calculate the synchronous speed of a 3-phase, 12-pole induction motor that is excited by a 60-Hz source. What is the nominal speed (*n*) if slip at full-load is 6%?

2. A 3-phase, 6-pole induction motor is connected to a 60-Hz supply. The voltage induced in the rotor bars is 4V when the rotor is locked (open-circuit voltage). If the motor turns in the same direction as flux, calculate the approximate voltage induced and its frequency at (i) 300 rpm, and (ii) 1000 rpm

3. The no-load current of a 75KW, 4000V, 3-phase, 900rpm, 60HZ induction motor is one-third and starting current is six times the full-load current. Calculate (i) the approximate values of full-load current, starting current, and no-load current (ii) the output torque, synchronous speed, and rotor power (air gap power) given that slip is 2%.

4. A 3-phase, 75 hp, 400 V induction motor has a full load efficiency of 91% and a power factor of 83% lagging. Calculate the nominal current per phase.

5. A 3-phase 5KV (L-L), 8-pole, 60Hz, squirrel-cage induction motor draws a current of 400A and total active power of 3.3255 MW when operating at full-load. Full-load speed is 890 rpm. Resistance of stator winding is 0.05Ω per phase and it is connected in wye. Total iron losses are 30KW and windage and friction losses are 20KW. Calculate: (i) Power factor at full load, (ii) Active power supplied to the rotor, (iii) Total copper losses of rotor (I^2R), (iv) Output power, (v) Torque, and (vi) Efficiency.

6. A 3-phase 8-pole induction motor is connected to a 3-phase system with line-to-line voltage of 400V (peak) and frequency of 60Hz. Stator resistor is 0.7Ω and total stator copper loss is 840W. Iron loss is 400W and friction loss is 133W. Motor is rotating at 888rpm. Input power factor is 95% lagging. Determine: (i) Input power (ii) Output mechanical power and efficiency.

7. A three-phase induction motor is connected to a 110V (rms, line-to-neutral), 60Hz source. Stator resistance is 0.1Ω per phase and total stator loss is 120W. Input power factor is 0.85 lagging. It is rotating with a speed of 650rpm. There are ten poles per phase. Total iron losses are 90W and frictional losses are 80W. Calculate: (i) Input power (ii) Power delivered to the rotor iii) Output power and efficiency.

Chapter 16 – Synchronous Generators

The primary way to produce electrical power for consumption is through *synchronous AC generators*. They can convert mechanical energy into electrical energy in excess of 1.5GW. The basic principle is the same as discussed under three-phase systems: a moving magnet creates a time-varying flux which links with the three-phase windings and induces voltage in them. For bigger synchronous generators with high power rating, magnetic field is produced by a set of moving magnetic poles, which makes the *rotor* of the machine, and three-phase windings in which voltage is induced are stationary and make *stator* of the machine. This is for the obvious reason of not using any moving parts (slip rings, carbon brushes etc.) between the load and the generator, since the generated voltage is very high, which produces a very large power when connected to the load. There may also be synchronous generators with stationary magnetic field and moving three-phase winding with one slip-ring connected to each phase. These machines are usually used for low-voltage and low-power generation. The three-phase synchronous generators with three-phase winding on the stator and magnetic flux produced by the rotor are also called *alternators.* Synchronous generators are called synchronous because the induced voltage frequency is synchronized (locked) to the speed of rotation of the magnetic field (rotor).

16.1 Number of Poles

The number of poles on the rotor in a synchronous generator depends upon the speed of rotation and the required induced voltage frequency. In general, the frequency of the induced voltage (in Hz) is given by the following equation:

$$f = \frac{np}{120} \tag{16.1}$$

where n is the speed of rotation of the rotor in rpm and p is the total number of poles. Hence, to produce a 60-Hz voltage waveform from a 2-pole (N-S) generator, it must be rotated at 3600rpm.

Example 16.1: It is required to produce the frequency of the induced voltage of a synchronous generator to be 60Hz using an 8-pole rotor. How fast the rotor should be rotated?

Solution:

f = 60Hz; *poles* = 8;

Since, $f = n*poles/120 \rightarrow n = 120*f/poles$ = 900 rpm

16.2 Features of the Stator

The stator is comprised of a cylindrical laminated core with a set of slots for the three-phase winding. The three-phase winding is generally lap-wound and always connected in wye with neutral connected to the ground.

The nominal line voltage of a synchronous generator depends upon its apparent power (KVA) rating. In general, the greater the power rating, the higher the voltage. However, the nominal line-to-line voltage rarely exceeds 25KV because the increased slot insulation takes up valuable space at the expense of copper conductors.

16.3 Features of the Rotor

There are two main types of rotors: *salient* and *non-salient*. Salient pole rotors have poles jutting out, as shown in *figure 16.1*.

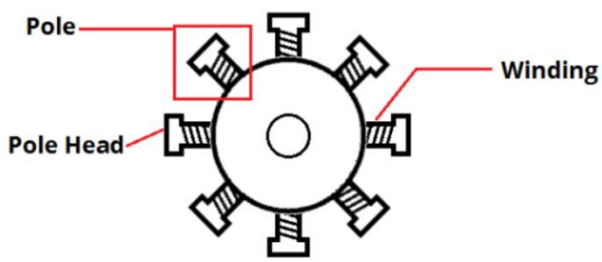


Figure 16.1: Salient Pole Rotor

Salient pole rotors are used with turbines that run at low speed, since, to produce 60-Hz frequency, more poles are required if the speed is low, per (16.1). Salient pole rotors can have many sets of poles since they are not physically connected to each other. One of the turbines where salient pole rotor is used is a hydropower turbine. In hydropower turbines, the diameter of the rotor is very large with many poles jutting out (salient pole rotor). The rotor is moved slowly from the waterfall to extract the maximum power and it is coupled directly to a waterwheel.

The other type of rotor is *non-salient*, which is also called *cylindrical* rotor. This type of rotor is generally used with high-speed turbines (e.g., steam turbines), since the number of poles on this type of rotor is very limited (usually two or four). *Figure 16.2* shows non-salient pole rotors with two and four poles.

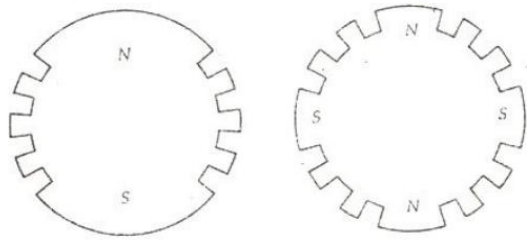


Figure 16.2: *Cylindrical Rotors*

Figure 16.3 shows cross-section of synchronous generators with both types of rotors.

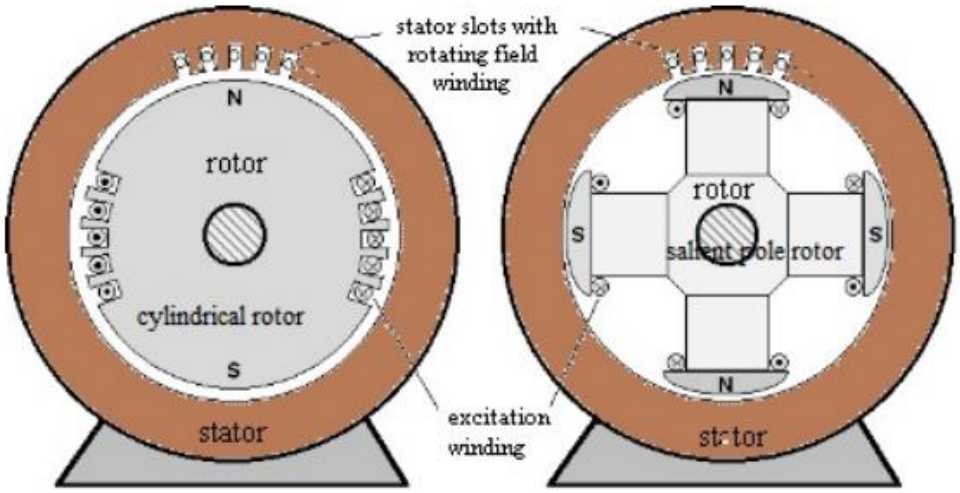


Figure 16.3: Synchronous Generators with Salient and Non-Salient Rotors

<u>*Example 16.2:*</u> *A waterfall is rotating a salient pole rotor with speed of 100 rpm. If the required output frequency is 60Hz, how many poles will be required on the rotor?*

<u>Solution:</u>

f = 60Hz; n = 100rpm;

Since, $f = n*poles/120 \rightarrow poles = 120*f/n =$ 72 or 36 pairs of N-S

16.4 Equivalent Circuit of an AC Generator

The synchronous generator has three-phase winding on the stator and the rotor is connected to a DC source supplying current to electromagnets to produce the magnetic flux, which is rotated inside the stator when a mechanical source rotates the rotor. A three-phase load is connected to the stator directly without any need of slip-rings and carbon brushes. *Figure 16.4* shows the electrical presentation of the generator and *figure 16.5* shows a three-phase load connected to a generator.

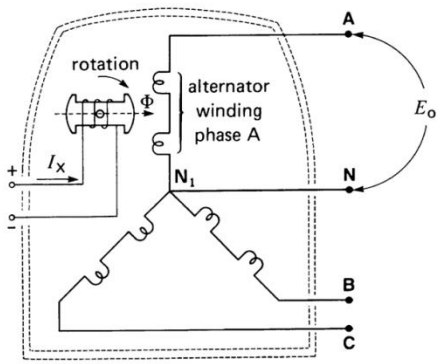


Figure 16.4: Electrical Presentation of a Synchronous Generator

As shown in *figure 16.4*, the electromagnet connected to a DC source and receiving an excitation current *Ix* is rotating clockwise and producing a flux ϕ, which is linked with the three-phase stator winding to produce a no-load phase voltage of E_o.

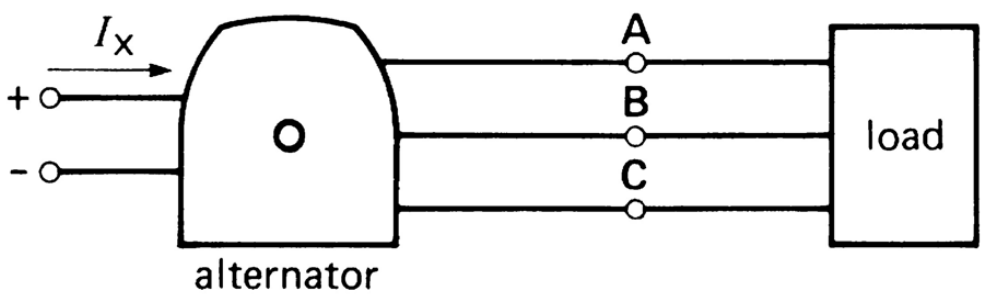


Figure 16.5: A 3-phase Load Connected to a Synchronous Generator

Each phase of the generator can be represented by the resistance and reactance of the winding, respectively called synchronous resistance and reactance. If the load is balanced and connected in a wye connection, the system can be represented as shown in *figure 16.6*.

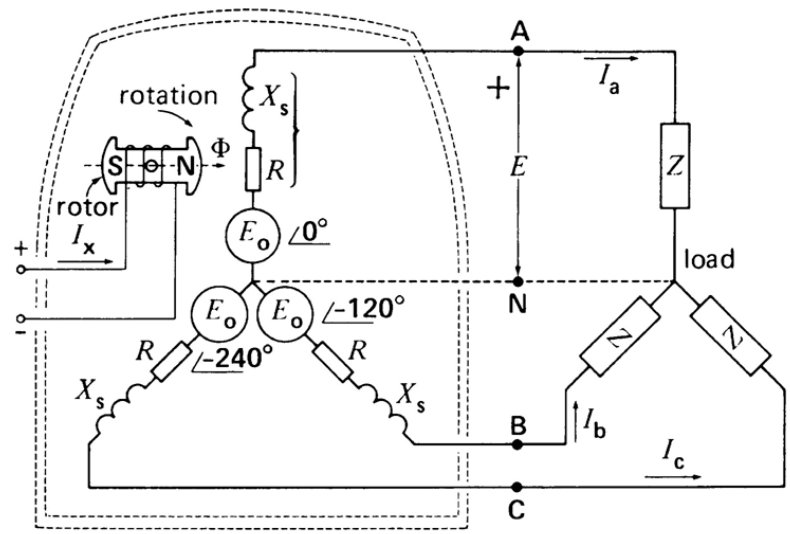


Figure 16.6: Three-phase Synchronous Generator with Phase Resistance and Reactance and Load

In general, the phase resistance is quite small as compared to the synchronous reactance; hence, we can safely ignore it to represent the equivalent circuit for each phase of the generator as shown in *figure 16.7*.

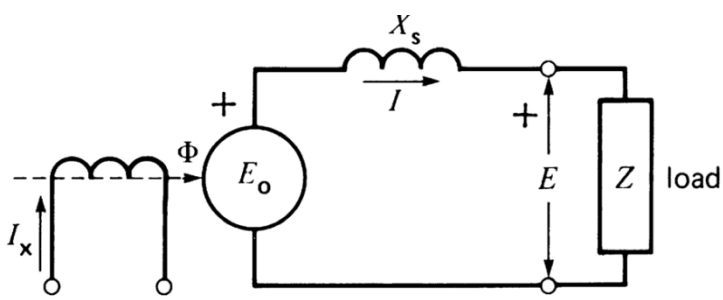


Figure 16.7: Single Phase of a Synchronous Generator with Line (or Phase) Reactance

16.5 <u>Synchronous Reactance</u>

To determine the value of the synchronous reactance, *open-circuit* and *short-circuit* tests are performed. In the open-circuit test, terminals of the generator are kept open and connected to a voltmeter. The rotor is rotated at the rated speed and field current is increased until the rated voltage is observed in the voltmeter connected between the terminals. Since terminals are opened, no current would flow through the windings of the stator and there is no voltage drop in the synchronous reactance. Therefore, the voltage observed at the terminals is in fact the rated induced voltage of the generator. The situation is similar to the circuit shown in figure *16.8* where the three

voltmeters are measuring the line-to-line terminal voltages. Although, the figure shows measurement of the line-to-line terminal voltage but to measure the induced phase voltage, line-to-neutral terminal voltage is used. Hence, if line-to-line is given, divide it by $\sqrt{3}$ to get line-to-neutral voltage.

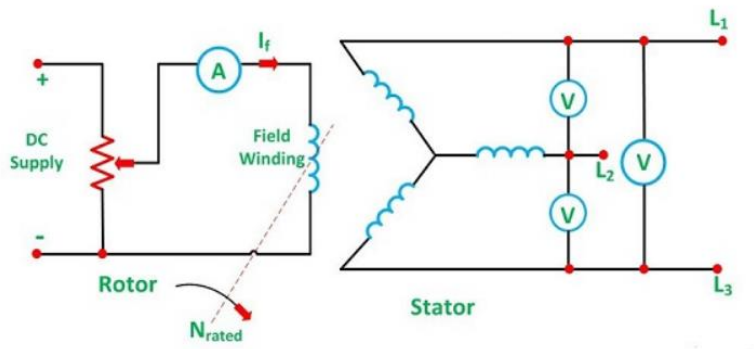


Figure 16.8: Open-Circuit Test to determine V_{oc}

Next, terminals of the generator are shorted out through an ammeter. The rotor is rotated at the rated speed and the same field current is given to the electromagnet in the rotor. This would produce the same flux as under the open-circuit test and the induced voltage would also be the same. The current passing through the ammeter is noted. This is shown in *figure 16.9*.

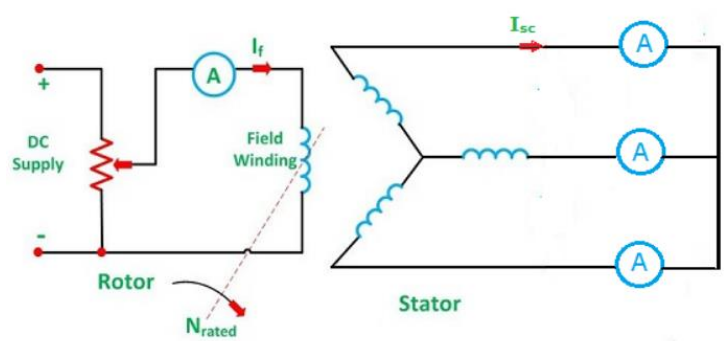


Figure 16.9: Short-Circuit Test to determine I_{sc}

The synchronous reactance can now be calculated by taking the ratio of the open-circuit voltage to the short-circuit current using Ohm's law:

$$Xs = \frac{V_{oc}}{I_{sc}} \tag{16.2}$$

<u>Example 16.3:</u> A 3-phase synchronous generator produces an open-circuit voltage of 7000V (LL) when field excitation current is 52A. The terminals of the generator are then shorted and three line currents are found to be 800A for the same excitation current. Calculate:

(i) Synchronous reactance Xs per phase (ii) If a balanced load of 5Ω resistors is connected to the terminals, determine the load current and voltage.

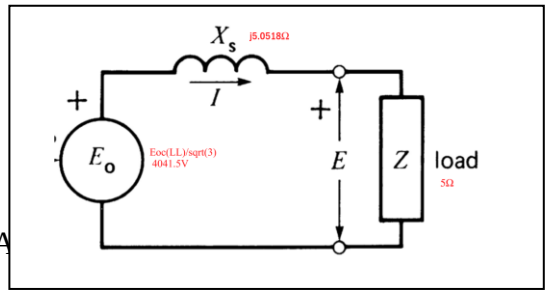
<u>Solution:</u>

(i) Synchronous reactance Xs per phase

>> Eoc_{LL} = 7000V; Isc = 800A; $Eoc_{LN} = Eoc_{LL}/sqrt(3)$ = 4041.5V;

Magnitude of the reactance: $Xs = Eoc_{LN}/Isc$ = 5.0581Ω

(ii) If a balanced load of 3Ω resistors is connected to the terminals, determine the load current and voltage.



>> Xs = j5.0581Ω; RL = 5Ω;

Current in the circuit: $I = Eoc_{LN}/(RL + Xs)$

= 399.48 - 404.12i A = 568.24∠-45.331°A

Load Current: $E = I*RL$ = 1997.4 - 2020.6i V

= 2841.2∠-45.331°V

16.6 <u>Synchronous Generator Under Load</u>

Two different types of loads can be connected to a synchronous generator: *isolated load*, and *infinite bus*.

Isolated Load: When a three-phase load (mostly balanced) is connected to the terminals of the generator, it is referred to as an isolated load.

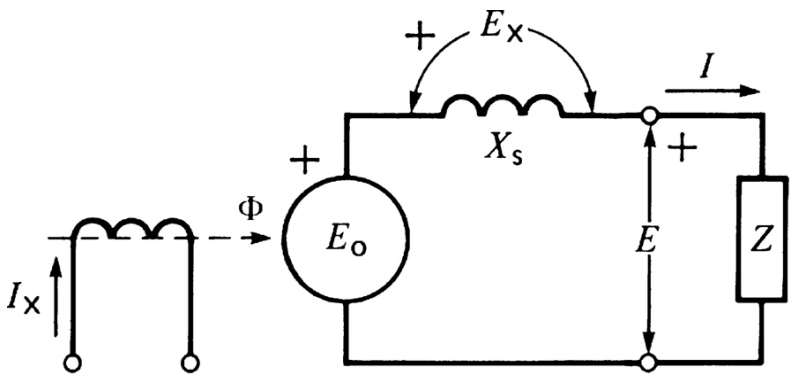


Figure 16.10: An Isolated Load Connected to a Synchronous Generator

Based on the type of the load, current *I* will either be lagging the induced voltage *Eo* (if the load is inductive) or leading it (if the load is capacitive). Most practical loads are inductive (e.g., motors), therefore, in general the current lags the induced voltage. The relationship between different voltages and currents, as shown in *figure 16.10*, is as follows:

(i) Load voltage *E* leads the load current *I* by an angle θ, where cos(θ) is the power factor of the load.

(ii) Load current *I* lags the voltage across the synchronous reactance, *Ex*, by 90°, which can be determined by Ohm's law, $\mathbf{E_x} = (jXs)(\mathbf{I}) \Rightarrow \mathbf{I} = \dfrac{\mathbf{E_x}}{jXs}$

(iii) Induced voltage *Eo* is the phasor sum of *Ex* and *E* (since, all of them are complex quantities)

Note that *Eo* and *Ex* are generator's internal voltages and can't be measured directly from the terminals if a load is connected. A phasor diagram for the circuit depicted in *figure 16.10* with an inductive load is shown in *figure 16.11*.

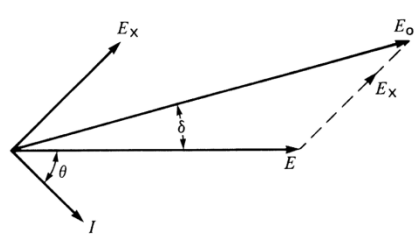


Figure 16.11: Phasor Diagram of Voltages and Current for an Isolated Inductive Load

If the isolated load is capacitive in nature, load current will lead the load voltage by an angle θ. *Ex* will still lead the load current, since *Xs* is pure inductance, and surprisingly, the magnitude of the

induced voltage will be smaller than the magnitude of the load voltage, although, the higher terminal voltage does not yield any more power. The corresponding phasor diagram is shown in *figure 16.12*.

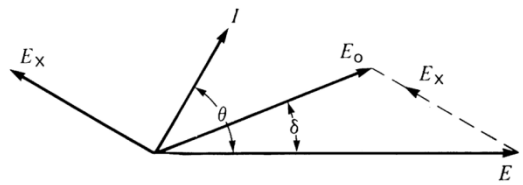


Figure 16.2: Phasor Diagram of Voltages and Current for an Isolated Capacitive Load

Infinite Bus: When a synchronous generator is connected to a power grid to which many other generators are also connected, the load is referred to as *infinite bus*. All generators connected to an infinite bus appear to be in parallel to each other. Infinite bus is such a powerful system that it imposes its voltage and frequency on any generator that is connected to it. When a generator is connected to an infinite bus, it becomes part of the power system and starts delivering power to thousands of loads connected to the power grid. Hence, it is impossible to specify the nature of a load connected to the terminals of any specific generator.

As soon as a generator is connected to the infinite bus, its terminal voltage assumes the bus voltage and induced voltage also becomes equal to the terminal voltage. Under this condition, no current flows from the generator towards the bus (Ohm's law) and it does not contribute to any power flow. The generator is said to be in a *floating* state.

Figure 16.3: Generator in the Floating State

To make the generator contribute power to the infinite bus, two parameters of the synchronous generator can be changed: field current, *Ix*, and *rotor speed*.

When field current, *Ix*, will increase, field flux will increase, which will result in the increase of the induced voltage *Eo*. Now, the current will start flowing from the generator to the infinite bus and there will be some voltage drop in *Xs*. However, since the synchronous reactance is much

smaller than the infinite bus load, therefore, both the induced voltage and the voltage across the infinite bus (terminal voltage) will be in phase and the current will be lagging all three voltages in the circuit by 90º, due to that fact that it would be lagging *Ex* by 90º (pure inductance) and *Ex* is in phase with *Eo* and *E*. The current can easily be calculated using Ohm's law ($I = \frac{\mathbf{Eo} - \mathbf{E}}{jX_s}$). This situation is shown in *figure 16.4*.

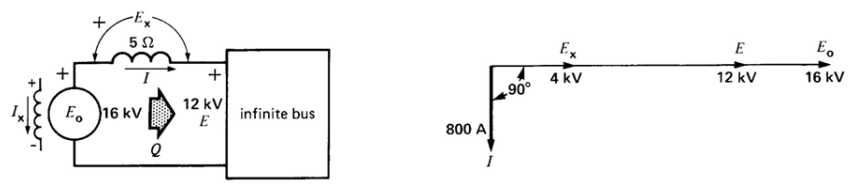


Figure 16.4: Over-excitation of the Generator

Once again, remember that all voltages and currents here are complex quantities, so do not attempt to perform mathematical operation on their magnitudes only.

Since, current is lagging the induced voltage, the only power flow in this case will be the reactive power flow and no real or active power will be delivered by the generator to the bus,

$$\begin{aligned} P &= 3E_o I\cos(90) = 0\,\text{W} \\ Q &= 3E_o I\sin(90) = 3E_o I\ \text{VAR} \end{aligned} \tag{16.3}$$

Under this condition, the generator will see the load as being inductive since current is lagging the induced voltage by 90º. This condition is called *over-excitation* of the generator.

If *Ix* is reduced below its rated value, flux will be reduced, and *Eo* will be decreased. Now, *Eo* will become less than the terminal voltage *E*. Hence, the current direction will be reversed; bus will start supplying current to the generator or the generator will see the load as a capacitive load. Current will lead both *Eo* and *E* by 90º but it will still lag *Ex* by 90º, since it is a pure inductor. The power flow in this case will still be reactive. The power, which is flowing from the bus to the generator, will partially magnetize the generator's field and rest will come from the field current *Ix*.

$$\begin{aligned} P &= 3E_o I\cos(-90) = 0\,\text{W} \\ Q &= 3E_o I\sin(-90) = -3E_o I\ \text{VAR} \end{aligned} \tag{16.4}$$

This condition is called *under-excitation* of the generator. The corresponding circuit and phasor diagram are shown in *figure 16.5*.

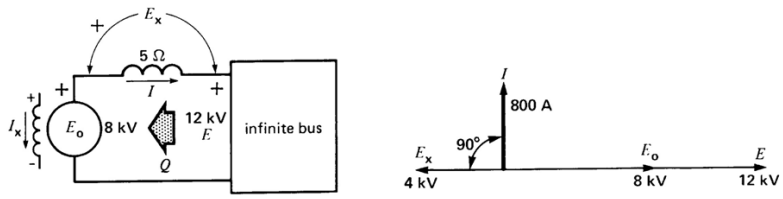


Figure 16.5: Under-excitation of the Generator

Under both over- and under-excitation conditions, no real power is delivered by the generator.

Now, let's discuss how a generator will deliver real or active power to the infinite bus. This happens when the speed of the rotor is increased by changing the mechanical torque of the turbine. The effect of increasing the mechanical torque of the turbine to increase the rotor speed is that the rated induced voltage will be achieved earlier than before, thus creating a phase difference between *Eo* and *E*, although their magnitudes will remain the same. The phase difference between *Eo* and *E* is called the *torque angle*, δ. Now, the generator will provide active power to the bus as well as some reactive power. The corresponding circuit and phasor diagram are shown in *figure 16.6*.

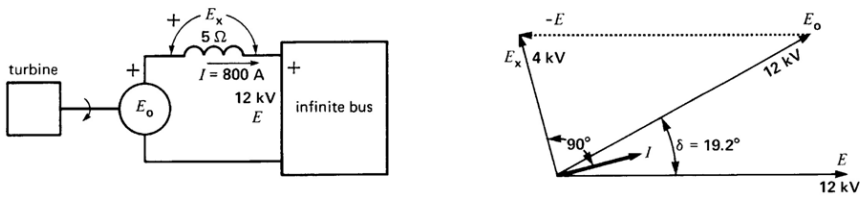


Figure 16.6: Torque of the Turbine is increased to deliver Real Power

If torque of the turbine keeps on increasing, torque angle will keep on increasing and generator will keep providing more active power to the infinite bus. When mechanical power of the turbine will become equal to the electrical power delivered by the generator to the bus, torque angle will become almost constant (instead of changing) and turbine will not accelerate anymore. At this time, the generator will be contributing a lot more active power to the bus as compared to the reactive power.

Once again, make sure not to get confused while doing calculation of different quantities in the circuit. All quantities are complex. For example, the magnitudes of the induced voltage and terminal voltage are same in *figure 16.6* but *Eo* also has a phase. Therefore, when calculating the current in the circuit, both magnitude and phase of each quantity must be considered.

If both field current and torque of the turbine increase simultaneously, the magnitude of *Eo* will increase as well as the torque angle. Both real and reactive powers will be supplied by the generator to the infinite bus under this condition.

<u>Example 16.4:</u> A 3-phase synchronous generator is connected to a power grid. The synchronous reactance (per phase) of the generator is 9Ω. If induced voltage is 12KV(LL), Calculate the active power that machine delivers when the torque angle is 30°.

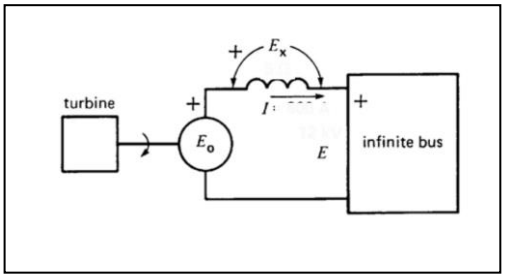
<u>Solution:</u>

Assume all values to be RMS.

>> $\delta = 30^o$; Xs = 9iΩ; $|Eo_{LN}|$ = 12000/sqrt(3)V = 6928.2V; $E = |Eo_{LN}|$

Eo_{LN} = 6928.2cos(δ) + *i* 6928.2sin(δ)= 6000.0 + 3464.1i V

$I = (Eo_{LN} - E)/Xs$ = 384.90 + 103.13i A



Total complex power delivered:

S = 3*Eo_{LN}*conj(I)

S = 8000000.00000 + 2143593.53945i VA

Therefore, total active power delivered is 8MW

PROBLEMS

1. A 3-phase generator possesses a synchronous reactance of 6Ω and the excitation (induced) voltage E_o is 3 KV per phase. Calculate line-to-neutral voltage E for a resistive load of 8Ω and draw the phasor diagram.

2. A 3-phase synchronous generator has excitation voltage Eo = 2.44KV, synchronous resistance Rs = 17Ω, and synchronous reactance Xs = 144Ω. Load per phase is 175Ω resistance. Calculate: (i) Line current, (ii) Line-to-Neutral voltage across the load, (iii) The power of the turbine driving the alternator (assume all the power of turbine is converted into the active power generated by the generator), (iv) Draw phasor between Eo and voltage across load.

3. A synchronous generator with synchronous reactance of 8Ω is connected to an infinite bus with voltage 10KV and frequency 60Hz. Both the field excitation current and speed of turbine is increased such that the excitation voltage is increased to 10.8KV with torque angle of 5°. Calculate the line current and active power delivered by the generator to the infinite bus.

4. A 3-phase synchronous generator is connected to an infinite bus with frequency 60Hz and line-to-line peak voltage of 2449.5V. The speed of the armature of the synchronous generator is increased to create a torque angle of 10°. Calculate the total active and reactive power supplied by the generator. Synchronous reactance is 5Ω.

5. The open-circuit voltage of a three-phase synchronous generator is 1KV (line-to-line, peak) and short terminal current is 100A (peak) for the same field current and rotor speed. If a 10Ω resistor is connected as a load, how much **total** active and reactive power will be supplied by the generator?

6. A 3-phase synchronous generator has an open-circuit line-to-neutral terminal voltage of 1kV (peak) and short-circuit line current of 444A (peak). The generator is connected to an infinite bus with line-to-neutral voltage of 1kV (peak). The speed of the generator turbine is increased to create a torque angle of 10°. Determine:
   (i) Total power drop in the synchronous reactance.
   (ii) Total power delivered to the infinite bus.

Chapter 25 – Transmission of Electrical Energy

Once electrical energy is generated through one of the many generation techniques (hydropower, wind, fossil fuels, solar, biofuels & waste, geothermal, chemical, nuclear, tidal, wave, etc.), the next step is its transmission and distribution. In this chapter, we will concentrate on the transmission of electrical energy, which generally happens at extra-high and high voltage, and mostly through overhead conductors. The generation and transmission system is usually 3-phase, whereas in the distribution phase, both three-phase systems (industries) and single-phase systems (households) are employed in U.S.A.

25.1 Principal Components of a Power Distribution System

An energy transmission and distribution system, under normal circumstances, must satisfy some basic requirements:

- Provides power to consumers without any interruption.
- Voltage must not vary by more than ±10%.
- Frequency must not vary by more than ±0.1Hz.
- Adheres with environmental and safety standards.
- Supplies energy at acceptable price to consumers.

Figure 25.1 shows a diagram with two generating stations connected with the transmission and distribution systems through multiple substations.

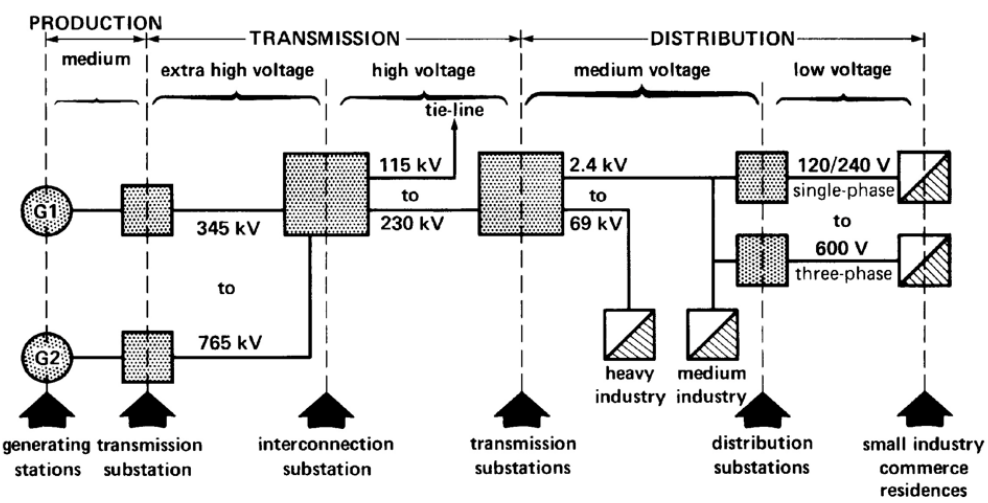


Figure 25.1: Block diagram of a Power System

Each transmission substation has transformers to step-up or step-down the incoming voltage and has means of voltage regulation. The distribution transformers have step-down transformers, which may also have automatic tap changing capability to regulate the low voltage. There are also interconnecting substations, which tie different power systems together to enable power exchanges between them, and to increase the stability of the overall network.

Some of the other important components of a substation are circuit breakers, fuses, and lightning arresters to protect expensive apparatus and to provide quick isolation of faulted lines from the system. Substations also have control equipment, disconnect switches, power measuring meters, capacitors, and inductors etc.

25.2 Types of Power Lines

Based on the active power that they carry and distance that they span, power lines are divided into *low voltage (LV), medium voltage (MV), high voltage (HV),* and *extra high voltage (EHV)*. Table 25.1 shows a number of standard voltages for transmission lines that are use in North America. All voltages are three-phase unless otherwise indicated. ☐ represents preferred voltages.

Table 25.1: Voltage Classes as Applied to Industrial and Commercial Power

	Nominal system voltage		
Voltage class	Two-wire	Three-wire	Four-wire
low voltage	120	120/240 ☐	—
	single phase	single phase	120/208 ☐
		480 V ☐	277/480 ☐
LV		600 V	347/600
medium voltage		2 400	
		4 160 ☐	
		4 800	
MV		6 900	
		13 800 ☐	7 200/12 470 ☐
		23 000	7 620/13 200 ☐
		34 500	7 970/13 800
		46 000	14 400/24 940 ☐
		69 000 ☐	19 920/34 500 ☐
high voltage		115 000 ☐	
		138 000 ☐	
		161 000	
HV		230 000 ☐	
extra-high voltage		345 000 ☐	
		500 000 ☐	
EHV		735 000–765 000 ☐	

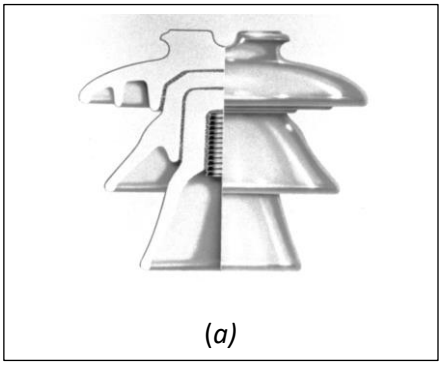
25.3 Components of a High-Voltage Transmission Line

There are three main components of an HV (or any other) transmission line; *conductors*, *insulators*, and *support structure*.

Conductors are always bare for HV lines. They are made out of copper or steel-enforced aluminum cable (ACSR), which are lighter and more economical. For longer lines, they are spliced, in which case, the joints are supposed to have low resistance and great mechanical strength.

Insulators are used to support and anchor the conductors and insulate them from ground. They are made of porcelain, but glass and other synthetic materials are also used. From an electrical point of view, insulators must have a very high resistance to surface leakage currents, and from a mechanical point of view, they must be strong enough to withstand the dynamic pull and weight of the conductors. There are two main types of insulators: *pin-type*, and *suspension-type*.

The pin type insulator has several porcelain folds, and the conductor is fixed at the top. *Figure 25.2* show the cross-sectional view and their use with power lines.



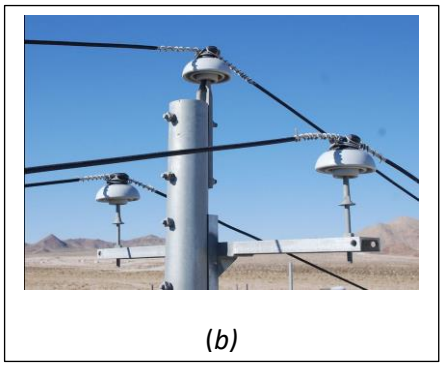
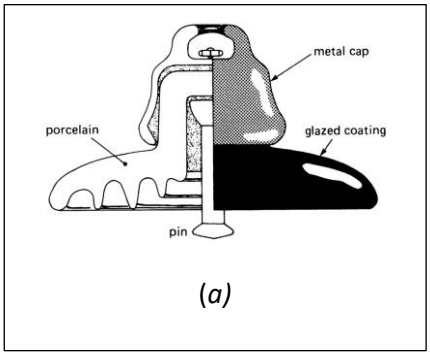


Figure 25.2: (a) Sectional view of a 69-kV pin type insulator (b)Pin-type insulator supporting power lines

For voltages above 70kV, suspension-type insulators are used, strung together by their pin and cap metallic parts. The number of insulators that are used depends upon the voltage of the line; for 110kV line, 3 to 6 insulators are used, for 230kV, 13 to 16, etc. *Figure 25.3* shows suspension-type insulators and their connection to power lines.



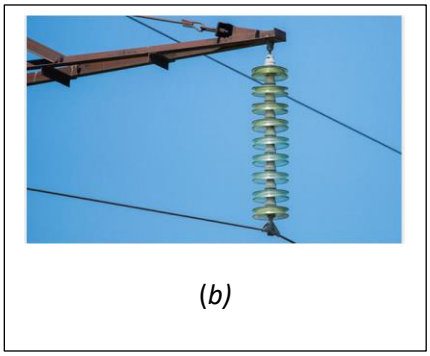


Figure 25.3: (a) Sectional view of a suspension-type insulator (b)Suspension-type insulators supporting power lines

A suspension-type insulator can also be used horizontally to isolate two lengths of wires from each other electrically while maintaining a mechanical connection, or where a wire attaches to a pole or tower, to transmit the pull of the wire to the support while insulating it electrically. In its horizontal use, it is called *strain* insulator. *Figure 25.4* shows strain insulators connecting to power lines.

Figure 25.4: Strain Insulators

Supporting structures are used to keep conductors at a safe height from the ground and at an adequate distance from each other. For voltages lower than 70kV, a single wooden pole with cross

arm can be used. For higher voltages, an H-frame wooden pole are used. For very high voltages, steel towers are used. *Figure 25.5* shows different supporting structures used in power systems.

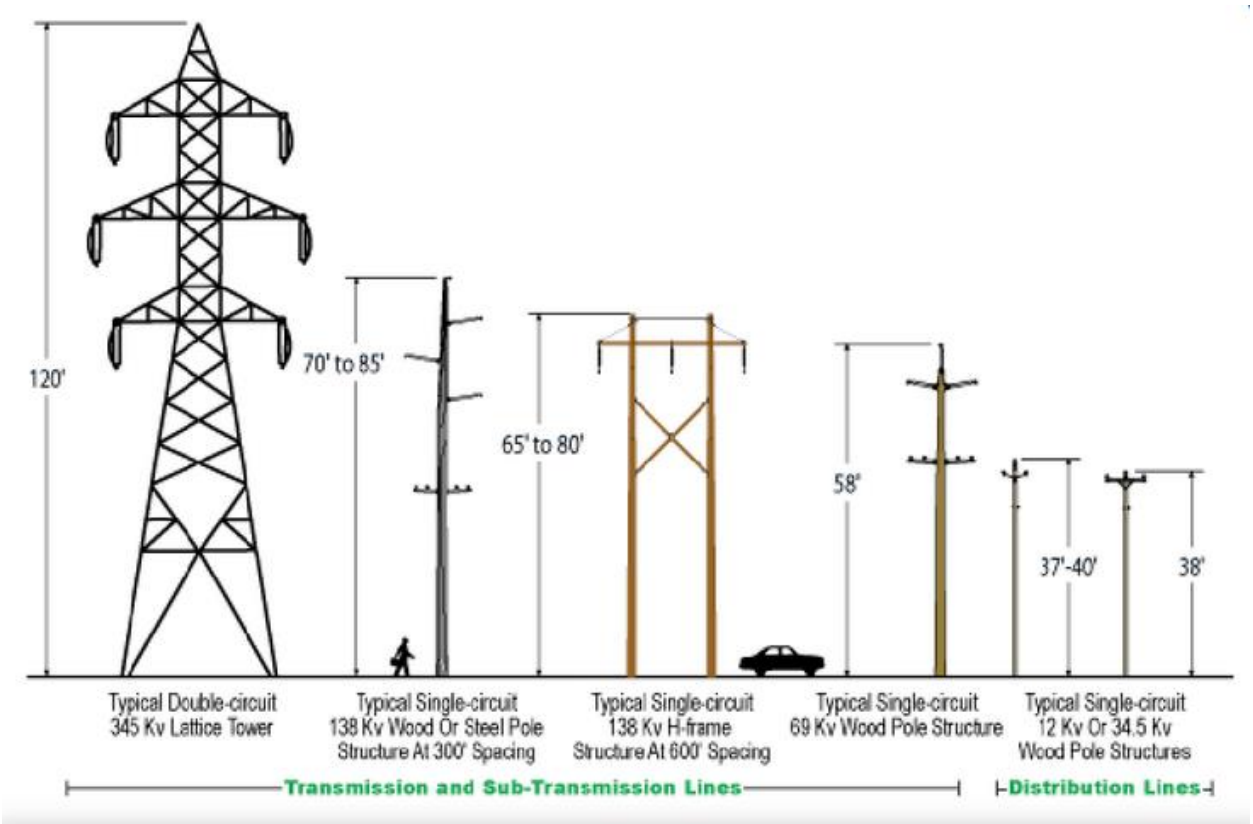


Figure 25.5: Different Supporting Structures used in Power systems

The spacing between conductors must be sufficient enough to prevent arc-over under gusty wind conditions. The spacing increases as the distance between towers increases. Also, there is more spacing between conductors for high voltage lines.

25.4 Construction of Transmission Lines

Once the conductor size, height and type of poles, and distance between two poles (span) is determined, a power line is stretched between two poles with an allowable *sag*. Sag is measured from the horizontal distance between the poles to the lowest point o the conductor. The tighter the line, the smaller the sag. Span and sag of a transmission line is are shown in *figure 25.6*

The permissible sag depends upon several factors; if it is summer and sag is not too much then during winter when lines will contract, the sag will reduce even more, and there may be a potential danger that the line will snap under mechanical strain. Similarly, if there is too much sag during wintertime, it will increase further during summer, and a potential danger may exist that lines may either touch each other or there may not be enough ground clearance.

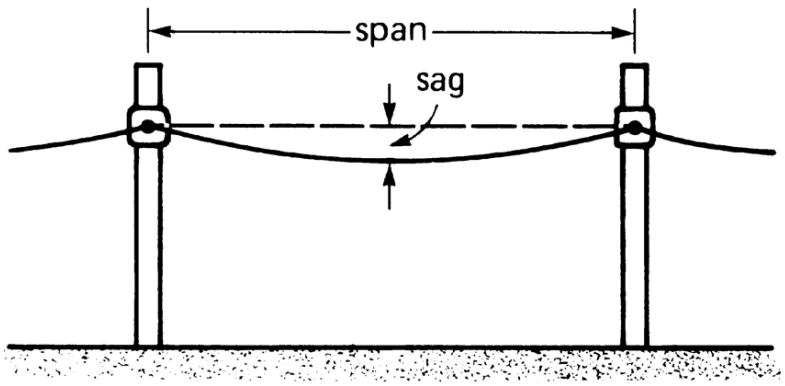


Figure 25.6: Span and Sag of a Transmission Line

During ice storms, lines may accumulate ice and become heavy. If the line is too tight, it may snap under the weight of ice. Under heavy wind, lines may begin to oscillate, and if the oscillation becomes too large, lines can snap or shorted out between phases. This condition is called *galloping lines*. To resolve this condition, lines are sometime equipped with mechanical weights to dampen the oscillations or to prevent their build-up altogether.

Insulators, over time, can accumulate salt, acid, dust etc., which can lead to deteriorate the insulation. They are required to be cleaned periodically to keep the system safe.

25.5 Corona Effect

Since air is not a perfect insulator, around very high voltage lines, air gets ionized which lead to *corona effect*. Corona effect can cause an audible hissing or cracking noise as it ionizes the air around the conductors. The corona effect can also produce a violet glow, production of ozone gas around the conductor, radio interference, and electrical power loss.

To reduce corona effect, instead of a single conductor, bundle of conductors may be used. Bundling conductors also have an added benefit of reduction of line inductance, which can lead to transmission lines carry more power. The diameter of the transmission line can also be increased to reduce corona effect.

25.6 Lightning Arresters

Lightning arresters are metallic rods that are placed at the highest points of tall structures (buildings etc.) and they are connected to ground electrode via conducting wires. The purpose of a lightning arrester is to carry high electrical energy that is conducted during a lightning event to the ground without channeling it through the important equipment, which can be destroyed. Since the resistance between the lightning arrestor and ground is very small, a very large current can flow through it during a lightning event. The instantaneous voltage of the system can reach to thousands of volts when the current is discharged and the system may pose an extreme danger if someone touches it during a lighting event.

25.7 Lightning and Transmission Lines

Although it is highly unlikely that lightning strikes the transmission line but if it does, it deposits a large amount of charge on the line, creating a large potential difference between the line and ground. High potential difference ionizes the surrounding air and results in electric arcing to discharge the additional charge. The whole process takes less than 50μs. Ionization of air acts as a short circuit that leads large amount of current flows from line to ground until circuit breakers trip and open the line. Speed of the fastest speed breaker is about $1/15^{th}$ of a second; 1000 times longer than the duration of lightning stroke.

Most of the time, lightning does not strike the phase but it strikes the ground conductor which is on the top and shields the line. The ground conductor is connected to ground at each tower. Generally, lightning arresters are installed on all lines going to a substation in order for the lightning not to destroy transformers and other equipment at the substation. Lightning arresters clip off the peak voltage travelling to a substation to a specific value to save equipment.

25.8 Fundamental Objectives of a Transmission Line

The fundamental purpose of a transmission or distribution line is to carry active power from one point to another. In addition to that, voltage should be kept almost constant along the length of the transmission line, which means that line losses must be kept as small as possible. These losses also contribute to heating of conductors (I^2R), therefore, less line losses means that lines will not be overheated.

25.9 Equivalent Circuit of a Line

Each transmission line has three components:

a. Resistance, over the length of the line,
b. Inductance, over the length of the line,
c. Capacitance, between line-line and line-neutral.

R and X_L increase with the increase in the length of transmission line, whereas, X_c decreases as it is modeled as being in parallel (capacitance adds up in parallel). Complete model of a phase of a transmission line is shown in *figure 25.7* where distributed impedances are shown along the line.

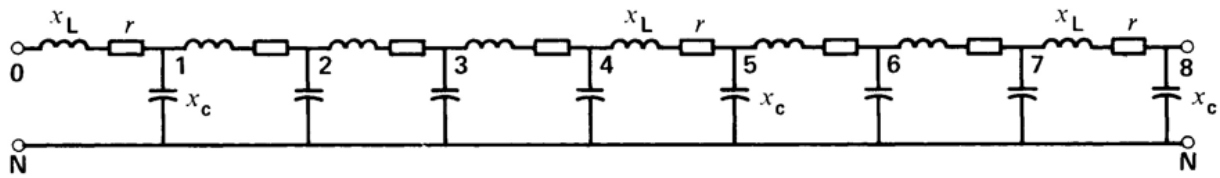


Figure 25.7: Distributed Impedance of a Transmision Line

An equivalent circuit of the transmission line can be created, as shown in *figure 25.8*, by adding up all resistances and inducatnces along the line and adding up capacitances in parallel. *Xc* is generally shown at the beginning and end of the transmission line.

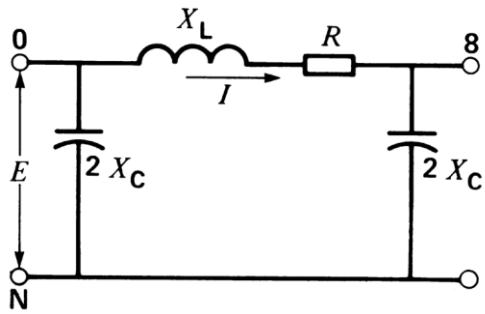


Figure 25.8: Equivalent Circuit of a Transmission Line

Resistance of the line depends upon frequency (skin effect), cross-section area of the wire, length of the wire, and resistivity of the conductor. *Table 25.2* shows resistance and ampacity (the maximum current that a conductor can carry continuously under the conditions of use without exceeding its temperature rating) of Copper and Aluminum Conductor Steel Reinforced (ACSR).

Table 25.2: Resistance and Ampacity of Some Bare Aerial Conductors

Conductor size		Resistance per conductor at 75°C		Ampacity in free air*	
AWG	Cross-section [mm^2]	Copper [Ω/km]	ACSR [Ω/km]	Copper [A]	ACSR [A]
10	5.3	3.9	6.7	70	–
7	10.6	2.0	3.3	110	–
4	21.1	0.91	1.7	180	140
1	42.4	0.50	0.90	270	200
3/0	85	0.25	0.47	420	300
300 kcmil	152	0.14	0.22	600	500
600 kcmil	304	0.072	0.11	950	750
1000 kcmil	507	0.045	0.065	1300	1050

Inductance and capacitance are function of frequency. As long as frequency of the system is not changing, inductive and capacitive reactance remain same per kilometer for overhead and underground lines. *Table 25.3* shows typical values of X_L and X_C for a 3-phase, 60-Hz system.

Table 25.3: Typical Impedance Values per km for a 3-Phase, 60-Hz System

Type of line	x_L [Ω]	x_C [Ω]
Aerial line	0.5	300 000
Underground cable	0.1	3 000

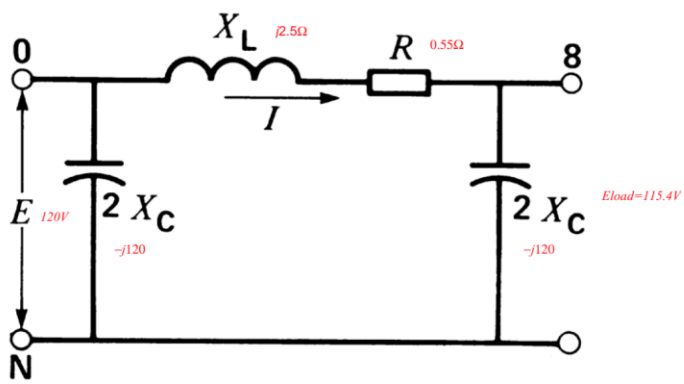
<u>Example 25.1:</u> A 3-phase, 60Hz, 208V (line voltage) overhead line has a length of 5km is composed of 5 Aluminum Cable Steel Reinforced (ACSR) conductors having a cross-section of 600 kilo-circular mills (kcmil). Line voltage at the load is 200V. Determine line resistance, inductive reactance, capacitive reactance, inductor, and capacitor values.

<u>Solution:</u> Using *Tables 25.2 and 25.3*

R = resistance of ACSR/km * distance = 0.11*5 = 0.55Ω

X_L = 0.5*5 = 2.5Ω → $L = X_L/(2\pi f)$ → 6.6mH

Xc = 300,000/5 = 60,000Ω → $C = 1/(2\pi fXc)$ = 44.2nF



One or more components of a transmission line can be eliminated to further simplify the circuit. Simplification depends on the relative magnitude of active and reactive powers, P_J, Q_L, and Q_C, associated with the line, compared to the active power delivered to the load, P. If any of the power associated with the line is negligible compared to the load power, that component may be ignored. *Figure 25.9* shows active and reactive powers of a transmission line, and power delivered to the load. Line powers can be calculated as follows:

$$\begin{aligned} P_J &= I^2 R \\ Q_L &= I^2 X_L \\ Q_C &= E^2 / Xc \end{aligned} \tag{25.1}$$

Where inductive reactive power is absorbed in the line and capacitive reactive power is generated by the line. To calculate capacitive reactive power, it is assumed that line and load voltages are same.

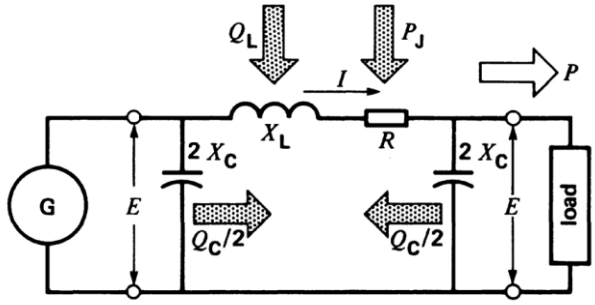


Figure 25.9: Active and Reactive Powers of a Transmission Line

Low voltage lines generally have shorth lengths for which capacitive reactance is high. Therefore, the reactive power associated with it, E^2/Xc, is small. Hence, Xc can be ignored for low voltage lines. The equivalent circuit is shown in *figure 25.10*.

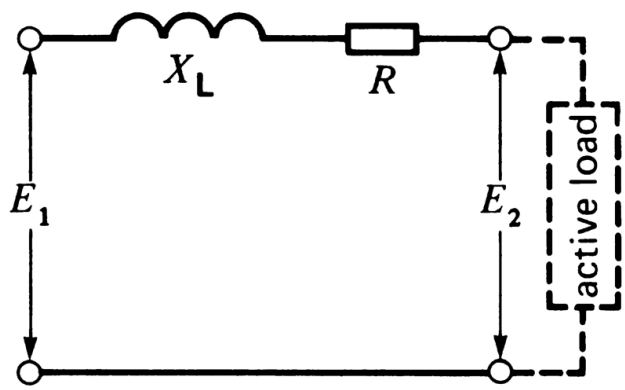


Figure 25.10: Equivalent Circuit of a Short LV Line

For very short distance, like house wiring, inductance can be neglected, only resistance can be considered.

Extra high voltage (EHV) lines that are very long, inductance and capacitance are more dominant as compared to resistance of the line. Therefore, I^2R losses are small and efficiency is high, and R can be ignored. Equivalent circuit is shown in *figure 25.11*.

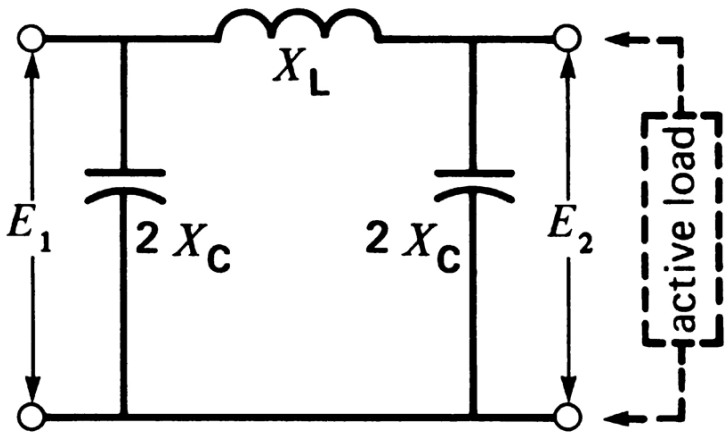


Figure 25.11: Equivalent Circuit of an EHV Line

Medium and high voltage lines can be represented by line inductive reactance only as, again, efficiency is high.

Example 25.2: A high voltage line has line voltage of 115KV and it spans for 30km. If it delivers 750KW to a 3-phase load, and load voltage is approximately the same as the line voltage, calculate line losses and suggest an approximate line circuit. Assume that the line is made up of ACSR with 600kcmil.

Solution:

First, equivalent line resistance, inductive reactance, and capacitive reactance will be calculated using *Tables 25.2* and *25.3*:

R = 0.11*30 = 3.3Ω

X_L = 0.5*30 = 15Ω

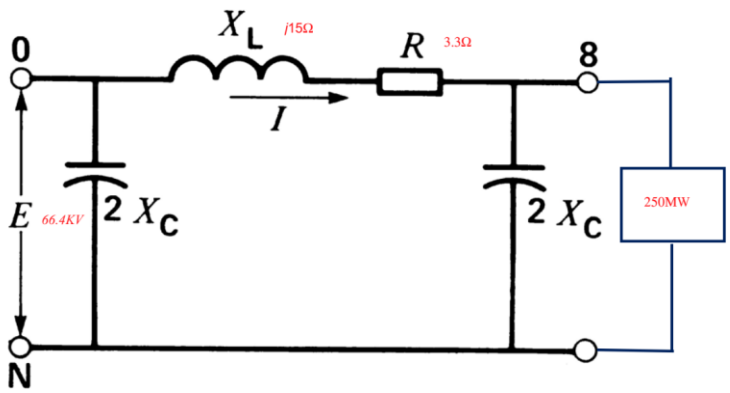
Xc = 300,000/30 = 10KΩ

Since Xc is too large, very small current will flow through it. Therefore, we can assume that the load current is approximately equal to the line current.

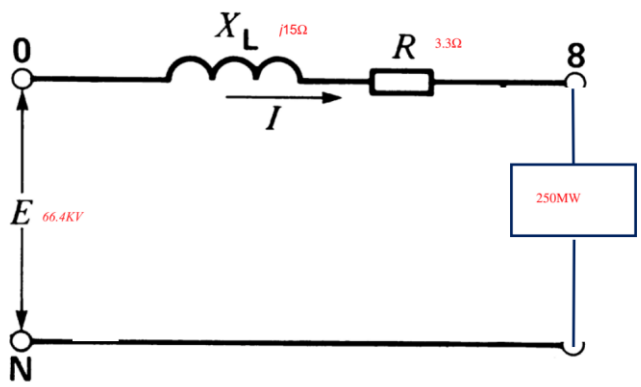
Load current: I = 250e6/(115000/sqrt(3)) = 3.7653KA

➔ P_J = I^2*3.3 = 46786389.41399 = 46.7864 MW
Q_L = I^2*15 = 212.6654 MVAR
Qc = (115e3/sqrt(3))^2/10e3 = 440.8333 KVAR = 0.440 MVAR

Since there is substantial line loss as compared to the power delivered to the load, it can't be ignored. Likewise, inductive losses can't be ignored. However, capacitive losses are small and can be ignored. Complete and approximate circuits are shown below.



Complete Equivalent Circuit



Approximate Circuit

Note that if the resistive line losses are very small as compared to the power delivered to the load, resistance can also be ignored and line will simply have the inductive reactance.

25.10 Voltage Regulation and Power Transmission Capability of Transmission Lines

Voltage regulation and power handling capacity are the two most important traits of a transmission line. Ideally, voltage should approximately be the constant from no-load to full load. However, around ±5% change in voltage from its nominal value is generally acceptable.

Active power that can be delivered from source to the load depends upon impedance of the line. Since only active power to the load is calculated, load will be modeled as a resistor. Four types of lines will be modeled and analyzed:

I. Resistive lines
II. Inductive lines
III. Inductive lines with capacitive compensation
IV. Inductive lines connecting two large systems

(i) *Resistive Lines:* The line is modeled as a resistor, as shown in *figure 25.12*

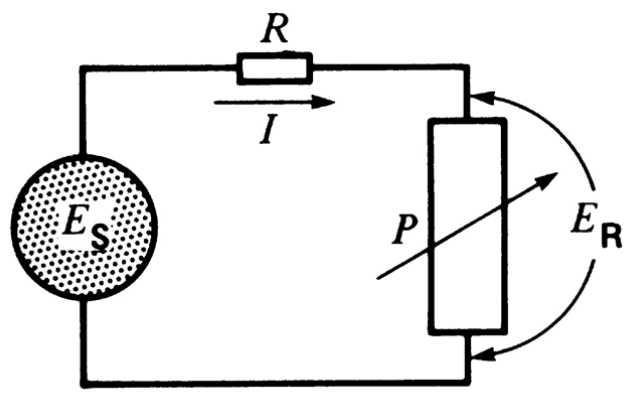


Figure 25.12: Resistive Line

The load is changed from open-circuit to a short circuit and receiver voltage E_R as well as power absorbed is observed. From Thevenin equivalent circuits, it is known that when the load resistor will be equal to the line resistance, load will receive the maximum power and E_R will be half of the source voltage. If load resistance is increased, load voltage will increase but the power delivered to the load will decrease. A characteristic curve between the receiver voltage and power absorbed is shown in *figure 25.13*.

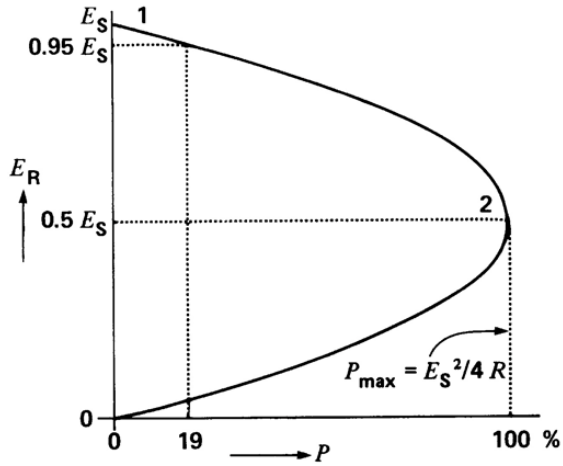


Figure 25.13: Characteristic Curve between Load Voltage and Load Resistance

If the load is changed such that is much larger than the line resistance, E_{load} will increase, however, the power received by the load will be much smaller than the peak power when load is equal to the line resistance.

Example 25.3: Assume that a 3-phase transmission line has resistance of 6.7Ω with line voltage of 208V (RMS). If the load resistance is 100KΩ, what percent of the peak power will be delivered to the load?

Solution:

Phase voltage: ELN = 208/sqrt(3) = 120.09V

Line current: I = ELN/(R_{line} + R_L) = 1.2008mA

Load power per phase: PL = I^2*RL = 0.1442 = 144.2mW

Maximum power delivered to the load will be when *RL* = *Rline* = 6.7Ω

Peak power per phase: Ppeak = (ELN/2)^2/Rline = 538.11W

Percentage of peak power: PL/Ppeak*100 = 0.026796%

Example 25.3: A single phase transmission line has 12Ω line resistance. If the supply voltage is 1kV (RMS), (i) what can be the maximum power supplied by the line? (ii) if voltage regulation is 5%, what will be the output power and its percentage with respect to the maximum output power?

Solution:

(i) Maximum power delivered by the source: Pmax = (Es/2)^2/Rline = 20.833kW

(ii) Voltage regulation: VR = (Es – Eload)/Es*100

➔ Eload = Es - VR*Es = 950V

Line current: I = (Es - Eload)/Rline = 4.1667A

Load power: Pload = Eload*I = 3958.3W

Percentage_Power = Pload/Pmax*100 = 19%